Republic of the Philippines
OFFICE OF THE PRESIDENT
COMMISSION ON HIGHER EDUCATION

Commission on Higher Education
OFFICIAL RELEASE
CHED Central Office
RECORDS SECTION
C.P. Garcia Ave., U.P. Diliman, Q.C.

CHED MEMORANDUM ORDER
No. 48
Series of 2017

SUBJECT: POLICIES, STANDARDS AND GUIDELINES FOR THE BACHELOR OF SCIENCE IN MATHEMATICS (BS MATH) AND BACHELOR OF SCIENCE IN APPLIED MATHEMATICS (BS APPLIED MATH) PROGRAMS

In accordance with the pertinent provisions of Republic Act (RA) No. 7722 otherwise known as the "Higher Education Act of 1994", in pursuance of an outcomes-based quality assurance system as advocated under CMO 46 s. 2012, in light of the addition of two years to basic education as provided by Republic Act (RA) 10533 otherwise known as the "Enhanced Basic Education Act of 2013", in view of the new General Education Curriculum, promulgated under CMO 20 s. 2013, and for the purpose of rationalizing mathematics education in the country by virtue of Commission en banc Resolution No. 231-2017 dated March 28, 2017, the following policies, standards and guidelines, revising CMO 19 s. 2007, are hereby adopted and promulgated by the Commission.

ARTICLE I
INTRODUCTION

Section 1. Rationale

Based on the Guidelines for the Implementation of CMO 46 s. 2012, this PSG implements the "shift to learning competency-based standards/outcomes-based education." It specifies the 'core competencies' expected of BS Mathematics and BS Applied Mathematics graduates "regardless of the type of HEI they graduate from." However, in "recognition of the spirit of outcomes-based education and...of the typology of HEIs," this PSG also provides "ample space for HEIs to innovate in the curriculum in line with the assessment of how best to achieve learning outcomes in their particular contexts and their respective missions ".

Specifically, CHED strongly advocates a shift from a teaching- or instruction-centered paradigm in higher education to one that is learner- or student-centered, within a lifelong learning framework. The learner- or student-centered paradigm shifts from a more input-oriented curricular design based on the description of course content, to outcomes-based education in which the course content is developed in terms of learning outcomes. In this paradigm, students are made aware of what they ought to know, understand and be able to do after completing a unit of study. Teaching and assessment are subsequently geared towards the acquisition of appropriate knowledge and skills and the building of student competencies.

On the other hand, teachers remain crucial to the learning process as catalyst and facilitators of learning. Learning environment such as laboratories, facilities, libraries, shape the learning experience of students and are deemed important since a good environment will enable the development and assessment of student learning competencies. The development and acquisition of the target learning competencies is the expected outcome of each academic program.

Revisions to the basic education curriculum (RA 10533), specifically the addition of senior high school (Grades 11 and 12), and the approval of the new CHED General Education Curriculum (CMO 20 s. 2013) necessitate additional revisions to the current Programs, Standards, and Guidelines for the BS Mathematics and BS Applied Mathematics curricula.

This document shall serve as a guide and standard for the transformation of the current Bachelor of Science in Mathematics (BS Math) and Bachelor of Science in Applied Mathematics (BS Applied Math) curricula (CMO No. 19 s. 2007) towards a learner- or student-centered approach, incorporating the provisions of the new General Education curriculum. The revised curricula take into account the College Readiness Standards prepared by the CHED for General Education and approved by the Commission en banc on 28 October 2011.

Each higher education institution (HEI) shall be given ample space to innovate the curriculum in line with their assessment of how best to achieve the set learning outcomes in their particular contexts and their respective missions. The main objective for each of the HEIs is the achievement of learning outcomes through different strategies.

ARTICLE II
AUTHORITY TO OPERATE

Section 2. Government Authority

All private higher education institutions (PHEIs) intending to offer Bachelor of Science in Mathematics (BS Math) and Bachelor of Science in Applied Mathematics (BS Applied Math) must first secure proper authority from the Commission in accordance with this PSG. All PHEIs with existing BS Math and BS Applied Math program are required to shift to an outcomes-based approach. State universities and colleges (SUCs), and local colleges and universities (LUCs) should likewise strictly adhere to the provisions in these policies and standards.

ARTICLE III
GENERAL PROVISIONS

Per Section 13 of R.A. 7722, the higher education institutions shall exercise academic freedom in its curricular offerings but must comply with the minimum requirements for specific academic programs, the general education distribution requirements and the specific professional courses.

Section 3. The Articles that follow give minimum standards and other requirements and prescriptions. The minimum standards are expressed as a minimum set of desired program outcomes that are given in Article IV, Section 6. The Commission designed a sample curriculum to attain such outcomes. This curriculum is shown in Article V, Section 9. The number of units of this curriculum is herein prescribed as the "minimum unit requirement" under Section 13 of RA 7722. In designing the curriculum the Commission employed curriculum mapping. Annex A exhibits a sample curriculum map.

Using a learner-centered/outcomes-based approach the Commission also determined appropriate curriculum delivery methods shown in Article V, Section 11. The sample course syllabi given in Article V, Section 12 show some of these methods.

Based on the curriculum and the means of its delivery, the Commission determined the physical resource requirements for the library, laboratories and other facilities and the human resource requirements in terms of administration and faculty. See Article VI.

Section 4. The HEIs are allowed to design curricula suited to their own contexts and missions provided that they can demonstrate that the same leads to the attainment of the required minimum set of outcomes. In the same vein, they have latitude in terms of curriculum delivery and in terms of specification and deployment of human and physical resources as long as they can show that the attainment of the program outcomes and satisfaction of program educational objectives can be assured by the alternative means they propose.

The HEIs can use the CHED Implementation Handbook for Outcomes-Based Education (OBE) and the Institutional Sustainability Assessment (ISA) as a guide in making their submissions. See Article VII, Section 19.

ARTICLE IV
PROGRAM SPECIFICATIONS

Section 5. Program Description

5.1 Degree Name

The degree program described herein shall be called Bachelor of Science in Mathematics (BS Math) or Bachelor of Science in Applied Mathematics (BS Applied Math).

5.2 Nature of the Field of Study

Mathematics is often described as the science of patterns. Mathematicians seek to discover, analyze and classify patterns in both abstract objects and natural phenomena. The traditional domains of study are quantity (arithmetic), structure (algebra), space (geometry) and change (analysis). Mathematics offers distinctive and powerful modes of thought such as abstraction, generalization, deduction, inference, use of symbols and the axiomatic method. Mathematical truth is established through logical analysis and proof. As a universal discipline it is rich in both theory and applications.

Mathematics is used as an essential tool in many fields, including the natural sciences, engineering, medicine, finance and the social sciences. Apart from being the language of the physical sciences, mathematics shares much in common with the former, notably in the exploration of logical consequences of assumptions. Mathematics is also regarded as an art, having an aesthetic and creative side. The special role of mathematics in education (being part of the curricula from primary school to college) is a consequence of its foundational nature and universal applicability.

Mathematicians engage in pure mathematics or mathematics for its own sake, without having, at least initially or intentionally, application or utility in mind. Applied mathematics, on the other hand, is the branch of mathematics concerned with application of mathematical theories and methods to other fields. Applied mathematicians are academics, researchers, or professionals who work on practical problems, often involving the formulation, analysis, and use of mathematical models. In turn, their work inspires and motivates new mathematical discoveries that may lead to the development of new mathematical disciplines, as in the case of operations research or game theory, or mathematics-based disciplines, such as statistics and finance. There is no clear line separating pure and applied mathematics.

5.3 Trends and Developments in Mathematics in the 21st Century

The legacy of classical mathematical theory, discovery of modern mathematical theories and techniques, and emergence of efficient computing methods, robust symbolic mathematical software and powerful computers, have broadened the landscape of mathematics and have led to many advancements in mathematics and science in general.

Mathematical theories and techniques have become essential in many areas, notably finance and the life sciences. Experimental and computational mathematics continue to grow in importance within mathematics. Computation, simulation and visualization are playing increasing roles in both science and mathematics. The overlap between applied mathematics and statistics and other decision sciences has become more significant, especially with the recognition of the stochastic nature of varied phenomena. Mathematical models and quantitative methods are increasingly being used in many fields, and new and powerful models are needed to address global problems and issues like climate change, disaster mitigation, risk management, food, water, and population.

5.4 Program Goals

The BS Math/ Applied Math graduates shall be equipped with enhanced mathematical and critical thinking skills. Graduates are expected to have developed deeper appreciation and understanding of the importance of mathematics in history and the modern world. They will be able do research or perform jobs that require analytical thinking and quantitative skills.

The program provides students with substantial exposure to the breadth and depth of mathematics, from classical to contemporary, and from theoretical to applied. The curriculum covers foundational courses in core areas of mathematics/applied mathematics as well as advanced courses that will help prepare graduates to pursue higher studies or work in a variety of fields.

5.5 Professions/careers/occupations for BS Math/ Applied Math graduates

Graduates of BS Math/ Applied Math often obtain jobs in education (teaching high school math courses or tertiary level elementary/service courses), statistics, actuarial science, operations research, risk management, business and economics, banking and finance, and computing and information technology.

5.6 Allied Fields

Mathematics/ applied mathematics is closely related to the fields of statistics, physics, computer science, and engineering.

Section 6. Program Outcomes

The minimum standards for the Bachelor of Science in Mathematics/ Bachelor of Science in Applied Mathematics program are expressed in the following minimum set of learning outcomes:

6.1 Common to all baccalaureate programs in all types of institutions

The graduates have the ability to:

a) Articulate the latest developments in their specific field of practice. (PQF level 6 descriptor)

b) Effectively communicate orally and in writing using both English and Filipino languages.
c) Work effectively and independently in multi-disciplinary and multi-cultural teams. (PQF level 6 descriptor)
d) Demonstrate professional, social, and ethical responsibility, especially in practicing intellectual property rights and sustainable development.
e) Preserve and promote "*Filipino historical and cultural heritage*"(based on RA 7722).

6.2 Common to the Science and Mathematics Discipline

f) Demonstrate broad and coherent knowledge and understanding in the core areas of physical and natural sciences.
g) Apply critical and problem solving skills using the scientific method.
h) Interpret relevant scientific data and make judgments that include reflection on relevant scientific and ethical issues.
i) Carry out basic mathematical and statistical computations and use appropriate technologies in the analysis of data.
j) Communicate information, ideas problems and solutions, both, orally and in writing, to other scientists, decision makers and the public.
k) Relate science and mathematics to the other disciplines.
l) Design and perform safe and responsible techniques and procedures in laboratory or field practices.
m) Critically evaluate input from others.
n) Appreciate the limitations and implications of science in everyday life.
o) Commit to the integrity of data.

6.3 Specific to BS Math/ BS Applied Math

p) Gain mastery in the core areas of mathematics: algebra, analysis, and geometry.
q) Demonstrate skills in pattern recognition, generalization, abstraction, critical analysis, synthesis, problem-solving and rigorous argument.
r) Develop an enhanced perception of the vitality and importance of mathematics in the modern world including inter-relationships within math and its connection to other disciplines.
s) Appreciate the concept and role of proof and reasoning and demonstrate knowledge in reading and writing mathematical proofs.
t) Make and evaluate mathematical conjectures and arguments and validate their own mathematical thinking.
u) Communicate mathematical ideas orally and in writing using clear and precise language.

6.4 Common to a horizontal type as defined in CMO 46 s 2012

- For professional institutions: a service orientation in one's profession
- For colleges: an ability to participate in various types of employment, development activities, and public discourses particularly in response to the needs of the communities one serves
- For universities: an ability to participate in the generation of new knowledge or in research and development projects

Graduates of state universities and colleges must, in addition, have the competencies to support "national, regional and local development plans" (RA 7722).

The HEIs, at its option, may adopt mission-related program outcomes that are not included in the minimum set.

Section 7. Sample Performance Indicators

Performance indicators (PIs) assist in the evaluation of student learning or the achievement of the program outcomes. These are demonstrable traits developed not only through the core or discipline-specific courses but also more importantly through their collective experiences.

To achieve the program outcomes, graduates of the BS Mathematics/ BS Applied Mathematics program are expected to possess a wide range of knowledge, values and skills. The performance indicators presented even for the baccalaureate and science and mathematics graduates are evaluated in the context of a BS Mathematics/ BS Applied Mathematics graduate.

Graduates of all Baccalaureate Programs

Program Outcomes	Performance Indicators
a) Articulate the latest developments in their specific field of practice.	• Participate in continuing education and professional development in the specific field of practice.
b) Effectively communicate orally and in writing using both the English/Filipino language.	• Demonstrate effective oral and written communication using both English and Filipino languages. • Exhibit adequate technical writing and oral communication abilities.
c) Work effectively in multi-disciplinary and multi-cultural teams.	• Work effectively as a member of multi-disciplinary and multi-cultural teams. • Display good judgment of people, actions and ideas and communicate them efficiently. • Demonstrate effective leadership, coordination and decision-making skills. • Demonstrate productive project management skills.
d) Demonstrate professional, social, and ethical responsibility, especially in practicing intellectual property rights.	• Articulate the contribution of one's profession to society and nation building. • Articulate the responsibilities of a Filipino citizen in relation to the rest of the world. • Demonstrate respect for intellectual property rights. • Explain professional knowledge and ethical responsibilities.
e) Preserve and promote Filipino historical and cultural heritage based on RA 7722.	• Articulate one's possible contributions to society and nation building.

Graduates of Science and Mathematics Programs

Program Outcomes	Performance Indicators
f) Demonstrate broad and coherent knowledge and understanding in the core areas of the physical and natural sciences and mathematics.	• Discuss extensively and articulate information in the core areas of science and mathematics.
g) Apply critical and problem solving skills using the scientific method.	• Employ problem-solving skills using the scientific method. • Demonstrate critical thinking skills in solving problems. • Apply scientific reasoning.
h) Interpret scientific data and reflect on relevant scientific and ethical issues.	• Recognize the importance of relevant scientific data. • Summarize information using reflection on important scientific and ethical issues.
i) Carry out basic mathematical and statistical computations and use appropriate technologies in the analysis of data.	• Perform appropriate suitable mathematical and statistical computations in data analysis.
j) Communicate information, ideas problems and solutions both, orally and in writing, to other scientists, decision makers and the public.	• Demonstrate technical writing and public speaking abilities. • Disseminate information, ideas, problems and solutions to fellow scientists, decision makers and the public. • Participate actively in scientific forum and public discussions.
k) Connect science and math to the other disciplines.	• Apply scientific advancements in ways that are meaningful to other disciplines. • Propose solutions to environmental problems based on interdisciplinary knowledge.
l) Design and perform techniques and procedures following safe and responsible laboratory or field practices.	• Practice responsible laboratory and field practices that follow proper techniques and procedures. • Demonstrate precision in making observations and in distinguishing differences between samples and events. • Employ appropriate and correct experimental design. • Follow industry standards and national laws.
m) Accepts and critically evaluates input from others.	• Discern significant inputs from other disciplines. • Critically evaluate data and information.
n) Appreciate the limitations and implications of science in everyday life.	• Acknowledge scientific facts as part of everyday life.
o) Commit to the integrity of data.	• Adhere to data integrity. • Report results and data as honestly as possible.

Graduates of BS Mathematics/ Applied Mathematics

Program Outcomes	Performance Indicators
p) Gain mastery in the core areas of mathematics: algebra, analysis, and geometry.	• Undertake an independent study of an unfamiliar topic and present an accurate and in-depth discussion of the results of the investigation both orally and in writing. • Represent a given problem by a mathematical model and use this to obtain a solution to the given problem.
q) Demonstrate skills in pattern recognition, generalization, abstraction, critical analysis, synthesis, problem-solving and rigorous argument.	• Apply the appropriate techniques in solving mathematical problems. • Break down a complicated problem into simpler parts • Adapt known methods and tools in solving new problems.
r) Develop and enhance perception of the vitality and importance of mathematics in the modern world including inter-relationship within math and its connection to other disciplines.	• Discuss the mathematical concepts behind well-known solutions to real-life problems. • Discuss important breakthroughs in the solution of real-world problems where mathematics played a significant role.
s) Appreciate the concept and role of proof and reasoning and demonstrate knowledge in reading and writing mathematical proofs.	• Submit a paper or thesis that contains proofs of mathematical statements based on rules of logic. • Assess the validity of the mathematical reasoning in the works of others and identify errors and gaps, if any.
t) Make and evaluate mathematical conjectures and arguments and validate their own mathematical thinking.	• Given a true mathematical statement, questions and investigates truth of the converse or inverse statements. • Able to propose conjectures, investigate their truth or falsity, and write rigorous proofs of the investigation. • Given a survey, expository or research paper, is able to recreate proofs and arguments contained in the paper, provide examples or give illustrations, and propose generalizations of results.
u) Communicate mathematical ideas orally and in writing using clear and precise language.	• Able to prepare a well-written research paper (thesis or special project paper) that organizes and presents a body of mathematics in a detailed, interesting and original manner. • Able to give an oral presentation of results of the research paper before peers and teachers.

ARTICLE V
CURRICULUM

Section 8. Curriculum Description

The curriculum for the BS Math/BS Applied Math program is built around a traditional base of foundational and core courses in the major areas of mathematics and applied mathematics with the inclusion of specialized courses in mathematics, applied mathematics, relevant disciplines, and emerging areas.

Since the mathematics department of different schools will have their particular strengths and orientation, there is a provision for elective courses that will allow for flexibility and accommodate the department's special interests. HEIs may offer courses beyond those specified in the recommended courses, according to their faculty expertise, institutional resources, and thrusts.

A BS Mathematics/ BS Applied Mathematics program offering a minor or specialization must include at least 15 units of relevant courses and electives for the specific area of specialization. Minors or specializations may include actuarial science, computing, operations research or statistics, among others. HEIs offering minors or specializations must possess the necessary faculty resources and facilities.

Based on the guidelines of the Mathematical Association of America's Committee on Undergraduate Programs in Mathematics, the following recommendations are given for designing the curricula for the BS Mathematics and BS Applied Mathematics programs:

8.1 Develop mathematical thinking and communication skills

Courses designed for mathematics/applied mathematics majors should ensure that students:

- Progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof;
- Gain experience in careful analysis of data;
- Become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.

8.2 Provide a broad view of the mathematical sciences

All majors should have significant experience working with ideas representing the breadth of the mathematical sciences. In particular, students should see a number of contrasting but complementary points of view:

- Continuous and discrete;
- Algebraic and geometric;
- Deterministic and stochastic; and
- Theoretical and applied.

Majors should understand that mathematics is an engaging field, rich in beauty, with powerful applications to other subjects, and a wide range of contemporary open questions.

8.3 Require study in depth

All majors should be required to:

- Study a single area in depth, drawing on ideas and tools from previous coursework and making connections, by completing two related courses or a year-long sequence at the upper level;
- Work on a senior-level project that requires them to analyze and create mathematical arguments and leads to a written and an oral report.

8.4 Develop skill with a variety of technological tools

All majors should have experiences with a variety of technological tools, such as computer algebra systems, visualization software, statistical packages, and computer programming languages

Section 9. Sample Curricula

9.1 Curriculum Components

The components of the BS Math/ Applied Math curriculum are listed in Table 1a and 1b together with the **minimum** number of units in each component.

Table 1a. Components of the BS Math curriculum and their corresponding units.

COMPONENTS	UNITS
a. General Education Curriculum	36
b. Core Courses	51
c. Non-math Foundational Courses	10
d. Electives	
Math Electives	9
Qualified Electives/Cognates	6
Free Electives	6
e. Thesis/Special Problem	3
f. Physical Education (PE)	8
g. National Service Training Program (NSTP)	6
Total	**135**

May also be chosen from the list of math electives with approval of program adviser.

+Any academic course in any discipline freely chosen by the student.

Table 1b. Components of the BS Applied Math curriculum and their corresponding units.

COMPONENTS	UNITS
a. General Education Curriculum	36
b. Core Courses	51
c. Non-math Foundational Courses	10
d. Electives	
Math Electives Qualified Electives/Cognates* Free Elective+	9 6 6
e. Thesis/Special Problem	3
h. Physical Education (PE)	8
i. National Service Training Program (NSTP)	6
Total	**135**

**May also be chosen from the list of math electives with approval of program adviser.*

+Any academic course in any discipline freely chosen by the student.

a. General Education (GE) Courses

CHED Memorandum Order No. 20 series of 2013 prescribes the set of courses comprising the General Education Program, consisting of eight (8) required core courses, three (3) GE elective courses and the legislated course on the Life and Works of Rizal. The list of GE courses is given in Table 2a and a suggested sequence is given in Table 2b. The GE courses may be taught in English or Filipino.

GE electives are courses that conform to the philosophy and goals of General Education as contained in CMO 20 s. 2013. Each elective course must apply an inter- or cross-disciplinary perspective and draw materials, cases or examples from Philippine realities and experiences. The electives must cover at least any two domains. For BS Mathematics and BS Applied Mathematics students, at least one GE elective must come from the Math, Science and Technology domain.

Table 2a. The GE courses and their corresponding units.

	Domain	**Required Courses**	Units
1	Social Sciences and Philosophy	Understanding the Self/ *Pag-unawa sa Sarili*	3
2		Readings in Philippine History/ *Mga Babasahin hinggil sa Kasaysayan ng Pilipinas*	3
3		The Contemporary World/ *Ang Kasalukuyang Daigdig*	3
4		Ethics/Etika	3
5	Math, Science and Technology	Mathematics in the Modern World/ *Matematika sa Makabagong Daigdig*	3
6		Science, Technology, and Society/ *Agham, Teknolohiya, at Lipunan*	3
7	Arts and Humanities	Purposive Communication/ *Malayuning Komunikasyon*	3
8		Art Appreciation/ *Pagpapahalaga sa Sining*	3
		Mandated Course	
9		Life and Works of Rizal / *Buhay at mga Likha ni Rizal*	3

		Elective GE Courses	
10	GE Electives*	GE Elective 1	3
11		GE Elective 2	3
12		GE Elective 3	3
		TOTAL	**36**

**To be taken from at least two domains; one GE elective should be an MST course. Remedial courses (such as algebra and trigonometry) cannot be credited as part of the GE or program electives.*

Table 2b. Suggested sequence of GE courses and mandated Rizal course.

	1st Semester	2nd Semester
Year 1	Mathematics in the Modern World	Readings in Philippine History
	Purposive Communication	Art Appreciation
	GE Elective 1	Understanding the Self
Year 2	GE Elective 2	Ethics
Year 3	The Contemporary World	Life and Works of Rizal
Year 4	Science, Technology, and Society	GE Elective 3

b. Core Courses (51 units)

The following core courses found in Tables 3a and 3b comprise the minimum requirements of the BS Math and BS Applied Math programs.

Table 3a. Core courses for the BS Mathematics program.

PROGRAM: BS MATHEMATICS	
DESCRIPTIVE TITLE	UNITS
a. Abstract Algebra I	3
b. Advanced Calculus I	3
c. Advanced Course in Analysis or Algebra⁺	3
d. Calculus I, II, III *	12 (4,4,4)
e. Complex Analysis	3
f. Differential Equations I	3
g. Fundamental Concepts of Mathematics	3
h. Fundamentals of Computing I	3
i. Linear Algebra	3
j. Modern Geometry	3
k. Numerical Analysis or Mathematical Modeling	3
l. Probability	3
m. Statistical Theory	3
n. Topology or Elementary Number Theory	3
TOTAL	**51**

⁺This course may be one of the following: Advanced Calculus II, Real Analysis, or Abstract Algebra II.
**Calculus I, II, III may be offered as a series of courses with a minimum 12 units provided all the topics in the recommended syllabi are covered.*

Table 3b. Core courses for the BS Applied Mathematics program.

PROGRAM: BS APPLIED MATHEMATICS	
DESCRIPTIVE TITLE	UNITS
a. Advanced Calculus I	3
b. Calculus I, II, III *	12 (4,4,4)
c. Differential Equations I	3
d. Discrete Mathematics	3
e. Fundamental Concepts of Mathematics	3
f. Fundamentals of Computing I	3
g. Fundamentals of Computing II	3
h. Linear Algebra	3
i. Mathematical Modeling	3
j. Numerical Analysis	3
k. Operations Research	3
l. Probability	3
m. Statistical Theory	3
n. Theory of Interest	3
TOTAL	**51**

** Calculus I, II, III may be offered as a series of courses with a minimum 12 units provided all the topics in the recommended syllabi are covered.*

c. Non-math Foundational Courses (10 units)

These refer to courses specific to a discipline outside mathematics that are not part of the general education program. HEIs may offer these courses to provide additional skills and foundation for advanced courses.

Table 4. List of non-math foundational courses

	Required Non-Math Foundational Courses	UNITS
1	General Physics I (Mechanics) with laboratory	4
2	Biology/General Chemistry I/General Physics II (with or without lab)	3
3	To be determined by HEI*	3
	TOTAL	**10**

**Such as additional/higher course in computing, statistics, communications, language, economics.*

d. Qualified Electives/Cognates (6 units)

Qualified electives or cognates are any academic courses offered in allied or relevant fields in the HEI chosen by a student and approved by the program adviser. Together with the math/applied math electives, these courses serve to incorporate a research focus or specialization to the student's program. In combination with or in lieu of qualified electives or cognates, the student, with approval of the program adviser may also choose from the list of math/applied math electives to satisfy this requirement. They comprise six (6) units of the curricula for the BS Math and BS Applied Math programs.

e. Mathematics Electives (9 units)

Electives may be chosen from the recommended list of math/applied math courses below (see Tables 5a and 5b).

Table 5a. List of recommended elective courses for the BS Math program.

PROGRAM: BS MATHEMATICS	
DESCRIPTIVE TITLE	UNITS
a. Abstract Algebra II	3
b. Actuarial Mathematics I	3
c. Actuarial Mathematics II	3
d. Advanced Calculus II	3
e. Algebraic Geometry	3
f. Differential Equations II	3
g. Differential Geometry	3
h. Discrete Mathematics	3
i. Dynamical Systems	3
j. Fundamentals of Computing II	3
k. Graph Theory and Applications	3
l. History and Development of Fundamental Ideas in Mathematics	3
m. Mathematical Biology	3
n. Mathematical Finance	3
o. Mathematical Modeling	3
p. Numerical Analysis	3
q. Operations Research I	3
r. Operations Research II	3
s. Partial Differential Equations	3
t. Projective Geometry	3
u. Real Analysis	3
v. Set Theory	3
w. Theory of Interest	3

Table 5b. List of recommended elective courses for the BS Applied Math program.

PROGRAM: BS APPLIED MATHEMATICS	
DESCRIPTIVE TITLE	UNITS
a. Actuarial Mathematics I	3
b. Actuarial Mathematics II	3
c. Applied Multivariate Analysis	3
d. Automata and Computability Theory	3
e. Computational Complexity	3
f. Convex Analysis	3
g. Data Structures and Algorithms	3
h. Differential Equations II	3
i. Dynamical Systems	3
j. Graph Theory and Applications	3
k. History and Development of Fundamental Ideas in Mathematics	3
l. Linear Models	3
m. Linear Programming	3
n. Mathematical Biology	3
o. Mathematical Finance	3
p. Nonlinear programming	3

q. Operations Research II	3
r. Partial Differential Equations	3
s. Real Analysis	3
t. Risk Theory	3
u. Sampling Theory	3
v. Simulation	3
w. Time Series Analysis	3
x. Theory of Databases	3

f. Free Electives (6 units)

Free electives are academic courses in any discipline offered in the HEI freely chosen by a student. They comprise six (6) units of the curricula for the BS Math and BS Applied Math programs.

g. Thesis or Special Problem (3 units)

Each student in the BS Mathematics or BS Applied Mathematics program is required to complete a 3-unit Thesis or Special Problem course. The course provides opportunities for students to conduct research on a mathematics topic that builds on areas covered by the core and elective courses.

The thesis/special problem involves activities that include independent reading from mathematical literature and other sources, as well as problem solving. The final paper should contain, organize and present a body of mathematics or a solution to a mathematical problem in a detailed, coherent and original manner.

9.2 Sample Program of Study

The sample program of study with the recommended sequence of courses is given in Tables 6a and 6b. Institutions may modify the curriculum to suit their particular requirements and thrusts. Certain courses may be offered during the summer.

Table 6a. Sample program of study for BS Math and recommended sequence of courses.

BS MATHEMATICS (135 units)								
	First Semester				**Second Semester**			
Year	**Descriptive Title**	**Units**			**Descriptive Title**	**Units**		
I								
	Calculus I	4		4	Calculus II	4		4
	Fundamentals of Computing 1	3		3	Fundamental Concepts of Math	3		3
	GE Course 1	3		3	GE Course 4	3		3
	GE Course 2	3		3	GE Course 5	3		3
	GE Course 3	3		3	GE Course 6	3		3
	PE I		2	0	PE II		2	0
	NSTP		3	0	NSTP		3	0
	Total	**16**	**5**	**21**	**Total**	**16**	**5**	**21**

BS MATHEMATICS (continued)								
Year	First Semester				Second Semester			
	Descriptive Title	Units			Descriptive Title	Units		
II								
	Calculus III	4		4	Advanced Calculus I	3		3
	Abstract Algebra I	3		3	Linear Algebra	3		3
	General Physics I Lec and Laboratory	3	1	4	Probability	3		3
	Non-math Foundational Course	3		3	Biology or General Physics II or General Chemistry I	3		3
	GE Course 7	3		3	GE Course 8	3		3
	PE III		2	2	PE IV		2	3
	Total	**16**	**3**	**19**	**Total**	**15**	**2**	**17**

Year	First Semester				Second Semester			
	Descriptive Title	Units			Descriptive Title	Units		
III								
	Advanced Course in Algebra or Analysis*	3		3	Modern Geometry	3		3
	Differential Equations I	3		3	Topology or Elementary Number Theory	3		3
	Math Elective 1	3		3	Numerical Analysis or Mathematical Modeling	3		3
	Statistical Theory	3		3	Math Elective 2	3		3
	GE Course 9	3		3	GE Course 10	3		3
	Total	**15**	**(0)**	**15**	**Total**	**15**	**(0)**	**15**

Year	First Semester				Second Semester			
	Descriptive Title	Units			Descriptive Title	Units		
IV								
	Complex Analysis	3		3	Qualified Elective / Cognate 2	3		3
	Math Elective 3	3		3	Free Elective 2	3		3
	Qualified Elective / Cognate 1	3		3	GE Course 12	3		3
	Free Elective 1	3		3	Thesis or Special Problem	3		3
	GE Course 11	3		3				
	Total	**15**	**(0)**	**15**	**Total**	**12**	**(0)**	**12**

May be one of the following: Advanced Calculus II, Real Analysis, or Abstract Algebra II

Note: GE courses include Life and Works of Rizal (mandated subject).

Table 6b. Sample program of study for BS Applied Math and recommended sequence.

BS APPLIED MATHEMATICS (135 units)								
Year	First Semester				Second Semester			
	Descriptive Title	Units			Descriptive Title	Units		
I	Calculus I	4		4	Calculus II	4		4
	Fundamentals of Computing 1	3		3	Fundamentals of Computing II	3		3
	GE Course 1	3		3	GE Course 4	3		3
	GE Course 2	3		3	GE Course 5	3		3
	GE Course 3	3		3	GE Course 6	3		3
	PE I		2	0	PE II		2	0
	NSTP		3	0	NSTP		3	0
	Total	**16**	**5**	**21**	**Total**	**16**	**5**	**21**
II	Calculus III	4		4	Differential Equations I	3		3
	Fundamental Concepts of Mathematics	3		3	Linear Algebra	3		3
	General Physics I Lec and Laboratory	3	1	4	Probability	3		3
	Non-math Foundational Course	3		3	Biology or General Physics II or General Chemistry I	3		3
	GE Course 7	3		3	GE Course 8	3		3
	PE III		2	0	PE IV		2	3
	Total	**16**	**3**	**19**	**Total**	**15**	**2**	**17**
III	Discrete Mathematics	3		3	Numerical Analysis	3		3
	Advanced Calculus I	3		3	Operations Research	3		3
	Math Elective 1	3		3	Theory of Interest	3		3
	Statistical Theory	3		3	Math Elective 2	3		3
	GE Course 9	3		3	GE Course 10	3		3
	Total	**15**	**(0)**	**15**	**Total**	**15**	**(0)**	**15**
IV	Mathematical Modeling	3		3	Qualified Elective / Cognate 2	3		3
	Math Elective 3	3		3	Free Elective 2	3		3
	Qualified Elective / Cognate 1	3		3	GE Course 12	3		3
	Free Elective 1	3		3	Thesis or Special Problem	3		3
	GE Course 11	3		3				
	Total	**15**	**(0)**	**15**	**Total**	**12**	**(0)**	**12**

May be one of the following: Advanced Calculus II, Real Analysis, or Abstract Algebra II

Note: GE courses include Life and Works of Rizal (mandated subject). NSTP and PE courses are not included in the total number of units.

Section 10. Curriculum Map and Course Map

Based on the required minimum set of program outcomes, the CHED has determined a program of study that leads to the attainment of the outcomes. This program of study specifies a set of courses sequenced based on flow of content, with each course having a specified title, description, course outcome and credit unit. For this purpose, a sample curriculum map is included as part of the PSG. It is a matrix of all courses and the minimum set of program outcomes showing which outcome each course addresses and in what way. The map also determines whether the outcomes are aligned with the curriculum.

Higher education institutions shall formulate its curriculum map based on its own set of program outcomes and courses. A sample curriculum map is given in Annex A.

Section 11. Sample Means of Curriculum Delivery

A range of instructional methods can be employed that can also become means of assessing outcomes. These include lecture and discussion, problem-solving, individual or group reports, problem-sets, computing and programming exercises, computer simulations and visualization. Suggested teaching strategies and assessment activities are indicated in the course syllabus of each course.

Section 12. Sample Syllabi for Core Mathematics Courses

The course specifications provided in this CMO in Annex B apply only to the core courses and indicate the minimum topics to be covered in each area. The HEIs shall formulate the syllabus for all the courses in their respective BS Math/ Applied Math program.

HEIs may follow their own course specification s in the implementation of the program but must not be less than those specified for major courses.

ARTICLE VI
REQUIRED RESOURCES

Section 13. Administration

The minimum qualifications for the head of the unit offering the degree program are the following:

13.1 Dean of the college/unit

The dean of a college/unit must be at least a master's degree holder in any of the disciplines for which the unit/college offers a program; and a holder of a valid certificate of registration and professional license, where applicable.

13.2 Head of the mathematics department/unit

The head of the unit/department must be at least a master's degree holder in the discipline for which the unit/department offers a program or in an allied field (cf. Article IV Section 5.6).

Section 14. Faculty

14.1 Qualification of faculty

a. Faculty teaching in a BS Mathematics/ BS Applied Mathematics program must be at least a master's degree holder in mathematics or in an allied field (cf. Section 5.5).

b. All undergraduate mathematics courses in the recommended program of study for the BS Mathematics/ BS Applied Mathematics program starting from the 2nd year must be taught by at least an MS degree holder in Mathematics/ Applied Mathematics. Specialized courses in the program (e.g. actuarial science, computing, operations research, and statistics) must be taught by at least an MS degree holder in the appropriate field, or by an expert with equivalent qualifications (e.g. Fellow / Associate of the Actuarial Society of the Philippines).

14.2 Full time faculty members

The institution shall maintain at least 50% of the faculty members teaching in the BS Mathematics/ Applied Mathematics program as full time.

14.3 Teaching load

Teaching load requirements for the BS Mathematics/ Applied Mathematics program shall be as follows:

a. Full time faculty members should not be assigned more than four (4) different courses/subjects within a semester.
b. In no instance should the aggregate teaching load of a faculty member exceed 30 units per semester (inclusive of overload and teaching loads in other schools).
c. Teaching hours per day should not exceed the equivalent of 6 lecture hours.

14.4 Faculty Development

The institution must have a system of faculty development. It should encourage the faculty to:

a. pursue graduate studies in mathematics/ applied mathematics especially at the PhD level;
b. undertake research activities and publish their research output;
c. give lectures and present papers in national/ international conferences, symposia and seminars; and,
d. attend seminars, symposia and conferences for continuing education.

The institution must provide opportunities and incentives such as:

a. tuition subsidy for graduate studies;
b. study leave with pay;
c. deloading to finish a thesis or to carry out research activities;
d. travel grants for academic development activities such as special skills training and attendance in national/ international conferences, symposia and seminars; and,
e. awards and recognition.

Section 15. Library

Library personnel, facilities and holdings should conform to existing requirements for libraries which are embodied in a separate CHED issuance.

The HEI is likewise encouraged to maintain journals and other non-print materials to aid the faculty and students in their academic work. CD-ROMs could complement a library's book collection but should not be considered as a replacement for the same.

Internet access is encouraged but should not be made a substitute for book holdings and/or on-line subscription to books and journals.

Libraries shall participate in inter-institutional activities and cooperative programs whereby resource sharing is encouraged.

Section 16. Laboratories and Classrooms

16.1 Laboratory requirements

The institution or unit should provide a computer laboratory or computing facilities that can be used by students for their research and computing requirements.

Laboratories should conform to existing requirements as specified by law (RA 6541, "The National Building Code of the Philippines" and Presidential Decree 856, "Code of Sanitation of the Philippines").

16.2 Classroom requirements

a. For lecture classes, ideal size is 30 students per class, maximum is 50.
b. For laboratory and research classes, class size shall be 20-25 students per class.
c. Special lectures with class size more than 50 may be allowed as long as the attendant facilities are provided.

16.3 Educational technology centers

The institution should provide facilities to allow preparation, presentation and viewing of audio-visual materials to support instruction.

ARTICLE VII
QUALITY ASSURANCE

Section 17. Assessment and Evaluation

The institution/department shall have in place a program assessment and evaluation system. The HEI must show this in their syllabi and catalogue. Institutions may refer to the CHED Implementation Handbook for Outcome-Based Education (OBE) and the Institutional Sustainability assessment (ISA) for guidance.

Section 18. Continuous Quality Improvement (CQI) Systems

The HEI shall maintain at all times a high standard of instruction and delivery through the establishment of a program level Continuous Quality Improvement system. Institution/department must show organizational and process plans, and implementation strategies. Institutions may refer to the CHED Implementation Handbook for Outcome-Based Education (OBE) and the Institutional Sustainability assessment (ISA) for guidance.

Section 19. CHED Monitoring and Evaluation

The CHED, in harmony with existing guidelines on monitoring and evaluation, shall conduct regular monitoring on the compliance of respective HEIs to this PSG. An outcomes-based assessment instrument shall be used during the conduct of monitoring and evaluation.

Using the CHED Implementation Handbook for OBE and ISA as reference, an HEI shall develop the following items to be submitted to CHED when they apply for a permit for a new program:

1. The complete set of program outcomes, including its proposed additional program outcomes.
2. Its proposed curriculum, and its justification including a curriculum map.
3. Proposed performance indicators for each outcome. Proposed measurement system for the level of attainment of each indicator.
4. Proposed outcomes-based syllabus for each course.
5. Proposed system of program assessment and evaluation
6. Proposed system of program Continuous Quality Improvement (CQI).

ARTICLE VIII
TRANSITORY, REPEALING AND EFFECTIVITY PROVISIONS

Section 20. Transitory Provision

All private HEIs, state universities and colleges (SUCs) and local universities and colleges (LUCs) with existing authorization to operate the Bachelor of Science in Mathematics and Bachelor of Science in Applied Mathematics programs are hereby given a period of three (3) years from the effectivity thereof to fully comply with the requirements in this CMO. However, the prescribed minimum curricular requirements in this CMO shall be implemented starting Academic Year 2018-2019.

Section 21 Repealing Clause

All CHED issuances, rules and regulations or parts thereof, which are inconsistent with the provisions of this CMO, are hereby repealed.

Section 22 Effectivity Clause

This CMO shall take effect fifteen (15) days after its publication in the Official Gazette, or in a newspaper of general circulation. This CMO shall be implemented beginning Academic Year 2018-2019.

Quezon City, Philippines, May 18 2017.

For the Commission,

PATRICIA B. LICUANAN, Ph.D.
Chairperson

Attachments:

Annex A – Curriculum Mapping
Annex B – Course Specifications
Annex C – Sample Examinations

ANNEX A. CURRICULUM MAPPING

BS MATH/APPLIED MATH PROGRAM OUTCOMES

At the end of this program, the students are expected to be able to:

A. Common to all programs in all types of schools

a) Engage in lifelong learning and understanding of the need to keep abreast of the developments in the specific field of practice. (PQF level 6 descriptor)
b) Communicate effectively thru oral and in writing using both English and Pilipino.
c) Perform effectively and independently in multi-disciplinary and multi-cultural teams. (PQF level 6 descriptor)
d) Recognize professional, social, and ethical responsibility.
e) Appreciate the "*Filipino historical and cultural heritage*" (based on RA 7722).

B. Common to the discipline

f) Demonstrate broad and coherent knowledge and understanding in the core areas of mathematics.
g) Apply analytical, critical and problem solving skills using the scientific method.
h) Interpret relevant scientific data and make judgments that include reflection on relevant scientific and ethical issues.
i) Carry out basic mathematical and statistical computations and use appropriate technologies in the analysis of data.
j) Communicate information, ideas problems and solutions both, orally and in writing, to other scientists, decision makers and the public.
k) Connect science and mathematics to the other disciplines.
l) Design and perform techniques and procedures following safe and responsible laboratory or field practices.
m) Accept and critically evaluate input from others.
n) Appreciate the limitations and implications of science in everyday life.
o) Commitment for the integrity of data.

C. Specific to BS Math/Applied Math

p) Gain mastery in the core areas of mathematics: algebra, analysis, geometry
q) Demonstrate skills in pattern recognition, generalization, abstraction, critical analysis, synthesis, problem-solving and rigorous argument
r) Develop an enhanced perception of the vitality and importance of mathematics in the modern world including inter-relationship within math and its connection to other disciplines
s) Appreciate the concept and role of proof and reasoning and demonstrate knowledge in reading and writing mathematical proofs
t) Make and evaluate mathematical conjectures and arguments and validate their own mathematical thinking
u) Communicate mathematical ideas orally and in writing using clear and precise language

BS MATH/APPLIED MATH

SAMPLE CURRICULUM MAP

COURSES	RELATIONSHIP OF COURSES TO PROGRAM OUTCOME																				
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
A. GE Program Courses																					
Understanding the Self/ *Pag-unawa sa Sarili*																					
Readings in Philippine History/ *Mga Babasahin hinggil sa Kasaysayan ng Pilipinas*																					
The Contemporary World/ *Ang Kasalukuyang Daigdig*																					
Ethics/Etika																					
Mathematics in the Modern World/ *Matematika sa Makabagong Daigdig*																					
Science, Technology, and Society/ *Agham, Teknolohiya, at Lipunan*																					
Purposive Communication/ *Malayuning Komunikasyon*																					
Art Appreciation/ *Pagpapahalaga sa Sining*																					
Life and Works of Rizal / *Buhay at mga Likha ni Rizal*																					
GE Elective 1																					
GE Elective 2																					
GE Elective 3																					
B. Others																					
P.E. 1, 2, 3, 4			*P*	*P*	*P*								*P*		*P*						
NSTP 1, 2			*P*	*P*	*P*								*P*		*P*						
C. Mathematics Core Courses																					
Abstract Algebra I						*I*	*P*		*P*	*P*						*P*	*P*	*P*	*P*	*P*	*P*
Advanced Calculus I						*I*	*P*		*P*		*P*						*P*	*P*	*P*	*P*	*P*
Advanced Course in Analysis/ Algebra						*I*	*P*		*P*		*P*						*P*	*P*	*P*	*P*	*P*
Calculus I, II, III						*I*	*P*	*I*	*P*	*P*	*I*					*P*	*P*	*P*	*I*	*I*	*P*
Complex Analysis						*P*	*P*		*P*	*P*			*P*	*P*		*P*	*P*	*P*	*P*	*P*	*P*
Differential Equations I						*P*	*P*	*I*	*P*	*P*	*I*					*P*	*P*	*P*	*I*	*I*	*P*
Elementary Number Theory						*I*	*P*		*P*	*P*						*P*	*P*	*P*	*P*	*P*	*P*
Fundamental Concepts of Mathematics						*I*	*I*		*I*				*I*			*I*	*I*		*I*		*I*
Fundamentals of Computing I							*P*		*P*	*P*	*I*					*P*	*P*	*P*			*P*
Linear Algebra						*P*	*P*		*P*	*P*	*P*		*P*	*P*		*P*	*P*	*P*	*P*	*P*	*P*
Modern Geometry						*I*	*P*		*P*		*I*						*P*	*I*	*P*	*P*	*P*

Numerical Analysis (or Mathematical Modeling)						*P*	*P*	*P*	*P*	*P*	*I*		*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*
Probability							*I*		*P*				*I*				*I*	*I*	*I*		*I*
Statistical Theory							*P*	*P*	*P*	*P*	*I*		*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*
Topology							*P*		*P*		*P*						*P*	*P*	*P*	*P*	*P*
D. Applied Mathematics Core Courses																					
Advanced Calculus I							*P*		*P*		*P*						*P*	*P*	*P*	*P*	*P*
Calculus I, II, III						*I*	*P*	*I*	*P*	*P*	*I*					*P*	*P*	*P*	*I*	*I*	*P*
Differential Equations I						*P*	*P*	*I*	*P*	*P*	*I*					*P*	*P*	*P*	*I*	*I*	*P*
Discrete Mathematics						*I*	*P*									*P*	*P*	*P*	*P*	*P*	*P*
Fundamental Concepts of Mathematics						*I*	*I*		*I*				*I*			*I*	*I*		*I*		*I*
Fundamentals of Computing I							*P*		*P*	*P*	*I*					*P*	*P*	*P*			*P*
Linear Algebra						*P*	*P*		*P*	*P*	*I*		*P*	*P*		*P*	*P*	*P*	*P*	*P*	*P*
Mathematical Modeling						*P*	*P*	*P*	*P*	*P*	*I*		*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*
Numerical Analysis						*P*	*P*	*P*	*P*	*P*	*I*		*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*
Operations Research I							*P*		*P*	*I*	*P*			*I*	*P*	*P*	*P*	*P*	*P*	*P*	*P*
Probability							*I*		*P*				*I*				*I*	*I*	*I*		*I*
Statistical Theory							*P*	*P*	*P*	*P*	*I*		*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*
Theory of Interest							*P*		*P*		*I*		*P*				*P*	*P*	*P*	*P*	*P*
E. Non-math Foundational Courses																					
Gen. Physics I (lecture and lab)						*P*	*P*	*P*	*P*	*P*	*P*		*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*	*P*
F. Elective Courses																					
G. Free Electives																					
H. Thesis or Special Problem																					
Thesis	*P*	*D*													*P*		*D*		*D*	*D*	*D*
Special Problem	*P*	*D*							*P*	*P*	*P*				*P*		*P*	*P*	*P*	*P*	*D*

I: *INTRODUCED – The student gets introduced to concepts/principles.*
P: *PRACTISED – The student practices the competencies with supervision.*
D: *DEMONSTRATED – The student practices the competencies across different settings with minimal supervision.*

ANNEX B. COURSE SPECIFICATIONS

BS Mathematics / Applied Mathematics

ABSTRACT ALGEBRA I

A. Course Details

COURSE NAME	**Abstract Algebra I**
COURSE DESCRIPTION	This course covers groups, subgroups, cyclic groups, permutation groups, abelian groups, normal subgroups, quotient groups and homomorphisms and isomorphism theorems, rings, integral domains, fields, ring homomorphisms, ideals, and field of quotients.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Fundamental Concepts of Mathematics

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
state with precision the definition of a group, a subgroup, a ring, a field, etc.						✓				✓						✓					✓
determine if a given set with given operation/s is a group, a subgroup, a ring, a field, etc.						✓	✓		✓	✓						✓	✓		✓	✓	
apply definitions and theorems to carry out computations and constructions involving different algebraic structures						✓	✓		✓	✓						✓	✓	✓	✓		
apply definitions and theorems to prove properties that are satisfied by all groups, subgroups, rings, etc						✓	✓		✓	✓						✓	✓		✓	✓	✓
recall the definitions and the basic properties of certain examples of groups, e.g. dihedral, symmetric, alternating						✓				✓						✓		✓			✓

recall the definition and basic properties of other objects such as homomorphism, isomorphism, kernel, direct product of groups, quotient groups, etc.						✓				✓						✓		✓			✓

C. Course Outline

Week	Topics
1-2	Preliminaries • Sets • Equivalence Relations • Functions • Binary Operations • Division Algorithm in Z and Modular Operations
3-4	Groups • Definition and elementary properties • Group tables • Order of a group • Subgroups • Isomorphism of groups
5-6	Cyclic Groups and Cosets • Definition, Order of an element • Structure of cyclic groups • Cosets • Lagrange's Theorem
7-8	Permutation Groups • Permutations • The symmetric and alternating groups • Dihedral group • Cayley's Theorem
9-10	Direct Product and Generating Sets • The direct product • Subgroup generated by a subset • Fundamental theorem of finitely generated abelian groups
11-12	Quotient Groups and Homomorphisms • Normal subgroup • Quotient group • Homomorphisms and basic properties • Isomorphism theorems
13-14	Rings • Definition and basic properties • Subring • The group of units of a ring • Ideal • Quotient ring
15-16	Ring Homomorphisms, Integral Domains, Fields • Basic properties of ring homomorphism • Ring isomorphism theorems • Zero divisors, integral domains • Fields • Field of quotients of an integral domain

D. Suggested Teaching Strategies

- Lectures, exercises, group discussion, individual inquiry

E. Suggested Assessment/Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. Textbooks/References:

- J. A. Gallian, Contemporary Abstract Algebra (7th ed.), Houghton Mifflin, 2010.
- J. Fraleigh, A First Course in Abstract Algebra (5th ed), Addison-Wesley, 2000.
- I. Herstein, Abstract Algebra *(2^{nd} ed)*, Collier Macmillan, 1990.
- T. Hungerford, Abstract Algebra, an Introduction *(2^{nd} ed)*, Saunders College, 1993

ABSTRACT ALGEBRA II

A. Course Details

COURSE NAME	**Abstract Algebra II**
COURSE DESCRIPTION	This course covers rings of polynomials, fundamental theorem of field theory, extension fields, algebraic extensions, finite fields, geometric constructions, fundamental theorem of Galois theory, illustrations of Galois theory.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Abstract Algebra I

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
calculate effectively in polynomial rings over various rings.									✓							✓	✓				
determine irreducibility of polynomials over a field using a variety of techniques.									✓							✓	✓		✓	✓	
determine whether an integral domain is a UFD.									✓							✓	✓		✓	✓	

explain connection between primes and irreducibles in arbitrary rings.									✓							✓	✓		✓	✓	
construct extension fields given an irreducible polynomial over the field.									✓							✓	✓				
determine the irreducible polynomial of an algebraic element over a field.									✓							✓	✓				
determine the index of a field in an extension field and a basis for the extension									✓							✓	✓		✓	✓	
give examples and non-examples of constructible real numbers.									✓							✓	✓		✓	✓	
describe the basic structure of finite fields and its subfields.									✓							✓	✓				
describe the splitting field and algebraic closure of a given field.									✓							✓	✓		✓	✓	
illustrate the Fundamental Theory of Galois Theory for small extensions.									✓							✓	✓		✓	✓	

C. Course Outline

Week	Topics
1	Introduction • Historical background • Solution of quadratic, cubic, quartic equations
2	Rings • Review of basic concepts on rings • Characteristic of a ring • Prime subfield • Prime ideal, maximal Ideal, principal ideal
3-4	Rings of Polynomials • Division algorithm in F[x] (F a field) • Ideal structure in F[x] • Divisibility conditions in ideal form • Irreducible polynomials • Tests for irreducibility
5-6	Factorization in Commutative Rings* • Unique factorization domains • Euclidean domains • Gaussian integers • Multiplicative norms

7-8	Extension Fields • Fundamental theorem of field theory (Kronecker's Theorem) • Algebraic and transcendental elements • Irreducible polynomial of an algebraic element • Extension fields as vector spaces
9	Finite Fields • Cyclic structure of group of units • Subfield structure • Frobenius automorphism
10-12	Special Extension Fields • Finite extensions • Algebraic extensions • Splitting fields • Algebraically closed fields, algebraic closure
13	Geometric Constructions • Constructible numbers • Trisecting an angle, doubling the cube
14	Some Important Theorems • Primitive element theorem • Isomorphism extension theorem
15-16	The Fundamental Theorem of Galois Theory* • The Galois group • The Galois correspondence (sketch of proof) • Normal extensions • Illustrations of Galois theory: finite fields, cyclotomic fields • *Insolvability of the quintic*

**If time permits. Italicized items are optional topics*

D. Suggested Teaching Strategies

- Lectures, exercises, group discussion

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Fraleigh. A First Course in Abstract Algebra
- Galllian. Contemporary Abstract Algebra
- Herstein. Abstract Algebra

ADVANCED CALCULUS I

A. Course Details

COURSE NAME	**Advanced Calculus I**
COURSE DESCRIPTION	Advanced Calculus I is the first of two courses that provides an introduction to mathematical analysis beyond the calculus series. Topics include the real number system, point set topology, limits and continuity, the derivatives, multivariable differential calculus, implicit functions and extremum problems.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Calculus III

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
prove completeness and topological properties of the real number system and R^n									✓							✓	✓		✓	✓	
prove convergence and divergence of a sequence of real numbers using the ϵ-δ definitions and theorems									✓							✓	✓		✓	✓	
identify and prove basic facts about continuity, derivatives, and their properties.									✓							✓	✓		✓	✓	
explain the differential for functions of one and several variables and apply to approximation									✓							✓	✓		✓	✓	
explain the Mean Value Theorem and its consequences									✓							✓	✓		✓	✓	
review of the technique of implicit differentiation for functions of a single variable and for functions of several variables.									✓							✓	✓		✓	✓	
investigate the validity of the technique and proof of the Implicit Function Theorem.									✓							✓	✓		✓	✓	
express the derivative and differential of a function as a matrix									✓							✓	✓		✓	✓	

C. Course Outline

Week	Topics
1	R as a Complete Ordered Field • Countable and uncountable sets
2-4	Point Set Topology • Euclidean space R^n • Open and closed sets in R^n • Accumulation points • Bolzano-Weiestrass Theorem • Heine-Borel Theorem • Compactness of R^n • Metric spaces • Compact subsets of a metric space • Boundary of a set
5-8	Limits and Continuity • Convergent sequences in a metric space • Cauchy sequences • Complete metric spaces • Limit of a function • Continuous functions • Continuity of composite functions • Examples of continuous functions • Continuity and inverse images of open or closed sets • Functions continuous on compact sets • Topological mappings • Uniform continuity and compact sets • Discontinuities of real-valued functions • Monotonic functions
9-11	Derivatives • Derivatives and continuity • The chain rule • One-sided derivatives • Rolle's theorem • The mean-value theorem for derivatives • Taylor's formula with remainder
12-14	Multivariable Differential Calculus • Rolle's theorem The directional derivative • Differential of functions of several variables • Jacobian matrix • The chain rule • Matrix form of chain rule • The mean-value theorem for differentiable functions • A sufficient condition for differentiability • A sufficient condition for equality of mixed partial derivatives • Taylor's formula for functions from R^n to R
15-16	Implicit Functions and Extremum Problems • Functions with nonzero Jacobian determinant • The inverse function theorem • The implicit function theorem • Extrema of real-valued functions of one variable • Extrema of real-valued functions of several variables

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Apostol. Mathematical Analysis
- Rudin. Principles of Mathematical Analysis
- Protter and Morrey. A First Course in Real Analysis
- Lang. Undergraduate Analysis
- Ross. Elementary Analysis: The Theory of Calculus

ADVANCED CALCULUS II

A. Course Details

COURSE NAME	**Advanced Calculus II**
COURSE DESCRIPTION	This course is a continuation of Advanced Calculus I. Topics include the convergence of sequences and series of real numbers, sequences and series of functions, uniform convergence, power series, functions of bounded variation and rectifiable curves, Riemann-Stieltjes integrals, interchanging of limit operations, multiple integration, improper integrals, transformations.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Advanced Calculus I

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
prove convergence and divergence of series of real numbers									✓							✓	✓		✓	✓	
define the Riemann integral on R and R^n using upper sums, lower sums, and/or limits									✓							✓	✓			✓	
use the definition to compute integral values in elementary cases									✓							✓	✓			✓	
identify sufficient and necessary conditions for									✓							✓	✓		✓	✓	

existence of the integral and prove theorems and properties of single and multiple integrals																					
explain Jacobian of a transformation and change of variable in multiple integrals.									✓							✓	✓			✓	
use transformations to "simplify" regions.									✓							✓	✓				
prove convergence and divergence of a sequence of real-valued functions									✓							✓	✓		✓	✓	
differentiate between pointwise and uniform convergence									✓							✓	✓		✓	✓	
prove convergence and properties of a series of functions									✓							✓	✓		✓	✓	
identify the interval of convergence of a power series									✓							✓	✓				
prove the Cauchy-Hadamard Theorem and explain its relevance									✓							✓	✓		✓	✓	
explain transformations and transformations defined implicitly by systems of equations									✓							✓	✓		✓	✓	
evaluate double integrals over more complicated regions of the plane.									✓							✓	✓				
define and give examples of vector and scalar fields, including directional derivatives, gradient, divergence and curl.									✓							✓	✓				
evaluate line integrals using definition and vector formulation									✓							✓	✓				
differentiate between path dependent and path independent line integrals									✓							✓	✓				
state and apply Green's theorem and Stoke's Theorem.									✓							✓	✓		✓	✓	

C. Course Outline

Week	Topics
1-3	Infinite Series • Limit superior and limit inferior of a sequence of real numbers • Infinite series • Alternating series • Absolute and conditional convergence • Tests for convergence of series • Dirichlet's test and Abel's test

	• Rearrangement of series • Double series and rearrangement theorem for double series • Multiplication of series
4-7	Riemann-Stieltjes Integral • Functions of bounded variation • Curves and paths • Rectifiable curves and arc length • Definition of Riemann-Stieltjes integral • Sufficient and necessary conditions for the existence of Riemann-Stieltjes integrals • Differentiation under the integral sign • Interchanging the order of integration • Multiple integrals and improper integrals
8-12	Sequences of Functions • Pointwise convergence of sequences of functions • Uniform convergence and continuity • Uniform convergence of infinite series of functions • Uniform convergence and Riemann-Stieltjes integration • Uniform convergence and differentiation • Power series
13	Green's Theorem for Rectangles and Regions
14	Review of Vector Fields
15-16	Surfaces • Surface area • Integrals over curves and surfaces • Stokes' Theorem, Gauss' Theorem

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Apostol. Mathematical Analysis
- Rudin. Principles of Mathematical Analysis
- Protter and Morrey. A First Course in Real Analysis
- Lang. Undergraduate Analysis
- Ross. Elementary Analysis: The Theory of Calculus

CALCULUS I

A. Course Details

COURSE NAME	**Calculus I**
COURSE DESCRIPTION	This is a first course in calculus. It covers limits, continuity, derivatives of algebraic and transcendental functions (exponential, logarithmic, trigonometric, hyperbolic and their inverses), applications of derivatives, differentials; antiderivatives, definite integrals, Fundamental Theorem of Calculus, and applications of definite integrals.
NUMBER OF UNITS	4 units (Lec)
PREREQUISITE	Pre Calculus Mathematics (Algebra and Trigonometry)

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
evaluate the limit of a function using the limit theorems.						✓	✓		✓	✓						✓	✓				
define continuity at a point and on an interval.						✓				✓						✓			✓		✓
distinguish between continuous and discontinuous functions						✓	✓			✓						✓			✓	✓	
use the definition to get the derivative of a function.						✓			✓	✓						✓					
apply the differentiation rules on various types of functions.						✓	✓		✓	✓						✓	✓				
apply the derivative tests to find maxima/minima of a function, graph functions and solve optimization problems.						✓	✓		✓	✓	✓					✓	✓	✓			
compute antiderivatives of various functions and definite integrals						✓	✓		✓	✓						✓	✓				
solve problems involving areas of regions, volumes of solids of revolution, arc lengths of curve and differential equations.						✓	✓		✓	✓	✓	✓				✓	✓				

C. Course Outline

Week	Topics
1-3	Limits and Continuity • Definition of limits and limit theorems (ε-δ definition optional) • One-sided limits, infinite limits, and limits at infinity • Continuity of a function and the Intermediate Value Theorem • The Squeeze Theorem and limits and continuity
4-7	Derivatives and Differentiation • The Derivative of a function • Formulas for differentiation of algebraic and transcendental functions • Chain Rule, implicit differentiation, higher-order derivatives • Indeterminate forms and L'Hopital's Rule • Increasing and decreasing functions and the 1st Derivative Test • Concavity and the 2nd Derivative Test • Sketching graphs of functions • Mean Value Theorem
8-11	Other Applications of Differentiation • Local linear approximation and differentials • Absolute extrema, Extreme Value Theorem, and optimization • Rectilinear motion • Related rates
12-16	Antiderivatives, Indefinite Integrals, and Applications • Antiderivatives and formulas of antidifferentiation • Integration by substitution • The definite integral • The Mean Value Theorem for integration • The Fundamental Theorem of Calculus • Area of a plane region • Arc length of a plane curve • Volumes by slicing, disks/washers, and cylindrical shells

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Anton, H., Bivens, I.C., and Davis, S., Calculus Early Transcendentals, 10th Edition, Wiley, *2011*.
- Anton, H., Bivens, I.C., and Davis, S., Calculus, 10th Edition, Wiley, *2012*.
- Edwards, Jr., C.H. and Penney, E., Calculus, Early Transcendentals, 7th Edition, Prentice Hall, 2007.
- Etgen, G., S. Salas and E. Hille, Calculus : One and Several Variables, 9th Ed., John Wiley and Sons, Inc., 2003.
- Leithold, Louis, The Calculus *7*, Harper Collins, 1996.
- Stewart, J., Calculus: Early Transcendentals, 7th Edition, Brooks/Cole, 2011.
- Thomas, G.B., Weir, M.D. and Hass, J.L., Thomas' Calculus, 12th Edition., Pearson, 2009.

- Thomas, G.B., Weir, M.D. and Hass, J.L., Thomas' Calculus Early Transcendentals, 12th Edition., Pearson, 2009.
- Varberg, D., Purcell, E.J., and Rigdon, S.E., Calculus Early Transcendentals, 1st Edition, Pearson, 2006.
- Varberg, D., Purcell, E.J., and Rigdon, S.E., Calculus, 9th Edition, Pearson, 2006

CALCULUS II

A. Course Details

COURSE NAME	Calculus II
COURSE DESCRIPTION	This course is the second of a series of three calculus courses. It covers techniques of integration, parametric equations and polar coordinates, cylindrical surfaces, surfaces of revolution, and quadric surfaces; vectors and vector-valued functions; functions of several variables; limits and continuity of functions of several variables; partial derivatives and the total differential; directional derivative; relative and absolute extrema of functions of several variables
NUMBER OF UNITS	4 units (Lec)
PREREQUISITE	Calculus I

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	PROGRAM OUTCOME																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
evaluate integrals using the basic techniques of integration						✓	✓		✓							✓	✓				
evaluate improper integrals						✓	✓		✓							✓	✓				
sketch graphs of equations in polar coordinates						✓	✓		✓							✓	✓				
identify and sketch graphs in space of lines, planes, cylindrical surfaces, surfaces of revolution and quadric surfaces						✓	✓		✓		✓					✓	✓	✓			
use calculus of vector-valued functions to analyze motion in space						✓	✓		✓		✓					✓	✓	✓			
evaluate limits and analyze the continuity of functions of several variables						✓	✓		✓							✓	✓				
find partial derivatives and directional derivatives of functions of several variables						✓			✓							✓					

solve problems involving the total differential						✓	✓		✓		✓					✓	✓	✓			✓
find relative and absolute extrema of functions of several variables						✓	✓		✓		✓					✓	✓				
apply the Lagrange multipliers method on constrained optimization problems						✓	✓		✓		✓					✓	✓	✓			✓

C. Course Outline

Week	Topics
	Techniques of Integration and Improper Integrals • Review of formulas of integration and integration by substitution • Integration by parts • Trigonometric integrals and integration by trigonometric substitution • Integration of rational functions by partial fractions • Improper integrals
	Parametric Curves, Polar Coordinates, and Surfaces • Parametric curves and the calculus of parametric curves • Polar coordinates and graphs of equations in polar coordinates • Tangent lines to, areas enclosed by, and arc length of polar curves • The three-dimensional Cartesian coordinate system • Cylindrical surfaces • Review of conic sections and quadric surfaces • Surfaces of revolution
	Vectors, Lines and Planes in Space, Vector-Valued Functions • Vectors in the plane and in space • Magnitude and direction angles • Vector operations • Dot and cross products of vectors • Scalar and vector projections • Lines and planes in space • Vector-valued functions • Calculus of vector-valued functions • Arc length and parametrization using arc length • Motion in space and normal and tangential components of acceleration • Curvature
	Differential Calculus of Functions of Several Variables • Functions of several variables, level curves, and level surfaces • Limits and continuity of functions of several variables • Partial derivatives • Higher-order derivatives, the total differential, and tangent plane approximation • The Chain Rule and implicit differentiation • Directional derivatives and gradients • Tangent planes to level surfaces • Relative extrema and the Second Derivatives Test • Absolute extrema and the method of Lagrange multipliers • Parametric surfaces and surfaces of revolution

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Same as Calculus I

CALCULUS III

A. Course Details

COURSE NAME	**Calculus III**
COURSE DESCRIPTION	This course covers sequences and series; double and triple integrals; applications of multiple integrals; vector fields; line and surface integrals.
NUMBER OF UNITS	4 units (Lec)
PREREQUISITE	Calculus II

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
apply the various tests for convergence and divergence of series;						✓	✓		✓							✓	✓		✓		
obtain the radius and interval of convergence of power series						✓			✓							✓					
evaluate double and triple integrals						✓			✓							✓					
solve problems involving applications of double and triple integrals						✓			✓							✓	✓				✓
evaluate line and surface integrals						✓			✓							✓					
solve problems involving applications of line and surface integrals						✓	✓		✓							✓	✓	✓			
define the curvature and geometry of plane and space curves						✓										✓					✓

define a vector field, its divergence, and curl						✓										✓					✓
perform a combination of gradient, divergence or curl operations on fields						✓	✓		✓							✓	✓				
evaluate line and surface integrals						✓	✓		✓							✓	✓				
state and apply Green's Theorem, Gauss' Divergence Theorem, and Stokes' Theorem						✓	✓		✓							✓		✓			✓
discuss the relationships between Green's Theorem, Gauss' Divergence Theorem, and Stokes' Theorem						✓				✓						✓					✓

C. Course Outline

Week	Topics
	Sequences and Series • Sequences • Series of constant terms and the n^{th}-term Test for Divergence • The Integral, Comparison, and Limit Comparison Tests • The Alternating Series, Ratio, and Root Tests • Power series and radius and interval of convergence of power series • Differentiation and integration of power series • Taylor, Maclaurin, and binomial series • Approximation using Taylor polynomials
	Multiple Integration • Double integrals • Double integrals in polar coordinates • Applications of double integrals (area, volume, mass, surface area) • Triple integrals • Triple integrals in cylindrical and spherical coordinates • Applications of triple integrals (volume, mass)
	Vector Fields and Line and Surface integrals • Vector fields • Curl and divergence • Line integrals of scalar and vector fields • The Fundamental Theorem of Line Integrals • Independence of path • Green's Theorem • Surface integrals of scalar and vector field • Stokes' Theorem and Gauss' Divergence Theorem

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Same as Calculus I and II

COMPLEX ANALYSIS

A. Course Details

COURSE NAME	**Complex Analysis**
COURSE DESCRIPTION	This course involves a study of the algebra of complex numbers, analytic functions, elementary complex functions, complex integration, and the residue theorem and its applications.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Advanced Calculus I

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
perform operations on complex numbers using the appropriate properties.						✓										✓	✓		✓		
use the appropriate tests to determine if a given function of a complex number is analytic.						✓	✓						✓			✓	✓	✓	✓	✓	✓
compare the properties of elementary functions of complex numbers with their real counterparts.						✓	✓						✓			✓	✓	✓	✓	✓	✓
use the appropriate theorems to evaluate the integral of a function of complex numbers.						✓	✓						✓			✓	✓		✓	✓	✓
represent a given analytic function by a specified series.						✓	✓				✓		✓			✓	✓		✓		✓
use the residue theorem to evaluate complex integrals and improper integrals.						✓	✓				✓		✓			✓	✓	✓	✓		✓

C. Course Outline

Week	Topics
1-2	The algebra of complex numbers • Cartesian, geometric and polar representations of complex numbers • Powers and roots • Stereographic projection
3-4	Functions of a complex variable • Limits and Continuity

	• Derivatives • Analytic functions and the Cauchy-Riemann equations in Cartesian and polar form • Harmonic functions
5-7	Elementary complex functions • Exponential functions and their properties • Complex trigonometric and hyperbolic functions • Complex logarithmic functions • Multiple valued functions and their branches • Complex exponents • Inverse trigonometric functions
8-9	Mappings of Elementary functions • Linear, reciprocal and linear fractional transformations • The power function and exponential function • Successive transformations
10-12	Complex Integration • Contours and Line Integrals • The Cauchy-Goursat Theorem • Cauchy's Integral Theorem and integral formula • Derivatives of functions • Morera's Theorem and the Fundamental Theorem of Algebra • Maximum moduli
13-15	Residues and Poles • Residues and the Residue Theorem • Laurent Series • The principal part of a function • Poles • Quotients of analytic functions • Improper integrals • Integration around a branch point

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Pennisi. Elements of Complex Variables
- Churchill, Brown, and Verhey. Complex Variables and Applications
- Lang. Complex Analysis
- Spiegel. Theory and Problems of Complex Variables

DIFFERENTIAL EQUATIONS I

A. Course Details

COURSE NAME	**Differential Equations I**
COURSE DESCRIPTION	This is an introductory course in ordinary differential equations (ODEs). It focuses primarily on techniques for finding explicit solutions to linear ODEs. Topics include first order ordinary differential equations, linear differential equations, linear equations with constant coefficients, nonhomogeneous equations, undetermined coefficients and variation of parameters, linear systems of equations; the existence and uniqueness of solutions.
NUMBER OF UNITS	3 units (Lec/Lab)
PREREQUISITE	Calculus III

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
solve ordinary differential equations by separation of variables, if applicable						✓	✓		✓	✓						✓	✓				
solve first-order ordinary differential equations						✓	✓		✓	✓						✓	✓				
solve second-order linear ordinary differential equations with constant coefficients and extend the technique to similar equations of higher order						✓	✓		✓	✓						✓	✓				
use Laplace transforms to solve linear ordinary differential equations and systems						✓	✓		✓	✓						✓	✓				
use the matrix exponential function to solve the linear system x'=Ax, where A is a 2x2 matrix with constant entries						✓	✓		✓	✓						✓	✓				
use qualitative analysis to sketch the solution curves of autonomous first-order ordinary differential equations						✓	✓		✓	✓						✓	✓				
use qualitative analysis to sketch the phase portrait of linear system x'=Ax, where A						✓	✓		✓	✓						✓	✓				

is a 2x2 matrix with constant entries																					
use linearization to describe the nature and stability properties of the fixed points of a non-linear autonomous system of order 2						✓	✓		✓	✓						✓	✓				
sketch the phase portrait of non-linear autonomous system of order 2 having a first integral						✓	✓		✓	✓						✓	✓				
submit a group report on an application of mathematical modelling using ordinary differential equations in economics, physics, engineering, or other areas.*						✓	✓	✓	✓	✓	✓					✓	✓	✓		✓	✓

**optional*

C. Course Outline

Week	Topics
1-2	Basic Denitions and Existence Theorems • Classification of ordinary differential equations and systems • Solutions and basic existence theorems • Direction field
3-7	Some Techniques of Solving Ordinary Differential Equations • Separation of variables • Leibniz's formula for first-order linear equations • Second-order linear ordinary differential equations with constant coefficients • Jordan canonical forms for 2 x 2 real matrices • The matrix exponential function; solutions of the linear system x'=Ax
8-13	Qualitative Analysis of Autonomous Differential Equation • First-order autonomous equations • Autonomous systems • Phase portraits of canonical linear systems in the plane • Phase portraits of non-canonical linear systems in the plane • Linearization at a fixed point • First integrals
14-16	Laplace Transforms • Definition and Examples • Properties of the Laplace Transform • Inverse Laplace Transform • Solving Initial Value Problems • Convolution

**optional*

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. Textbooks:

- Rainville, E.D., Bedient, P.E., and Bedient, R.E., Elementary Differential Equations, 8th Edition, Pearson, 1996.
- Edwards, C.H. and Penney, D.E. Elementary Differential Equations, 6th Edition, Pearson, 2007.
- Edwards, C.H. and Penney, D.E. Elementary Differential Equations with Boundary Value Problems, 6th Edition, Pearson, 2007.
- Polking, J., Boggess, A., and Arnold, D. Differential Equations and Boundary Value Problems, 2nd Edition, Pearson, 2005.
- Polking. J., Ordinary Differential Equations using Matlab, 3rd Edition, Pearson, 2003.
- Zill, D.G, Advanced Engineering Mathematics, 4th Edition, Jones and Bartlett, 2011
- Blanchard, Differential Equations, Thomson/Brooks/Cole, 2007
- Arrowsmith, D.K. & Place, C.M., Dynamical Systems, Chapman & Hall, 1992
- Coddington, E.A. & Levinson, N., Theory of Ordinary Differential Equations, McGraw-Hill, 1976
- Nagle, R.K., Saff, E.B., and Snider, A.D., Fundamentals of Differential Equations and Boundary Value Problems, Addison-Wesley, 2000
- Verhulst, F., Nonlinear Differential Equations and Dynamical Systems, Springer-Verlag, 2000.

DISCRETE MATHEMATICS

A. Course Details

COURSE NAME	**Discrete Mathematics**
COURSE DESCRIPTION	This is a course that covers the fundamentals of logic, proving, functions and sets, basic counting techniques, and advanced counting techniques.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Precalculus

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
Translate mathematical statements from common English to formal logic and vice-versa						✓	✓			✓						✓		✓			✓

Verify the validity of an argument using rules of inference						✓	✓			✓						✓	✓	✓	✓	✓	✓
Identify the difference among the various types of proof: direct proof, proof by contraposition, proof by contradiction, and proof by cases; and use an appropriate method in proving mathematical statements.						✓	✓			✓						✓	✓		✓	✓	✓
Use the proper notations on sets and functions and perform operations on them						✓										✓		✓			
Apply the basic and advanced counting techniques to solve counting problems						✓	✓		✓	✓	✓					✓	✓	✓			
Solve problems involving recurrence relations, generating functions and inclusion-exclusion principle						✓	✓		✓	✓	✓					✓	✓	✓	✓	✓	✓

C. Course Outline

Week	Topics
1-3	Propositional Logic • Propositional Equivalences • Predicates and Quantifiers • Nested Quantifiers • Rules of Inference
4-7	Introduction to Proofs • Proof Methods and Strategy • Sets • Set Operations • Functions
8-11	Counting • The Basics of Counting • The Pigeonhole Principle • Permutations and Combinations • Binomial Coefficients • Generalized Permutations and Combinations
12-15	Advanced Counting Techniques • Recurrence Relations • Solving Linear Recurrence Relations • Generating Functions • Inclusion-Exclusion Principle

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Rosen, K.H., Discrete Mathematics and Applications, 6th Edition, McGraw-Hill, 2007.
- Grimaldi. R.P., Discrete and Combinatorial Mathematics, 5th Edition, Pearson, 2003.
- Ross, K.A., Discrete Mathematics, 5th Edition, Pearson, 2002.
- Johnsonbaugh. R., Discrete Mathematics, 7th Edition, Pearson, 2007.

ELEMENTARY NUMBER THEORY

A. Course Details

COURSE NAME	**Elementary Number Theory**
COURSE DESCRIPTION	Properties of integers; divisibility; primes and unique factorization; solutions of congruences and residue systems; linear Diophantine equations, primitive roots; quadratic reciprocity law.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Fundamental Concepts of Mathematics

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
Understand the logic and methods behind the major proofs in Number Theory						✓	✓									✓	✓		✓	✓	✓
Construct mathematical proofs of statements and find counterexamples to false statements in number theory						✓	✓		✓								✓		✓	✓	✓
collect and use numerical data to form conjectures about the integers								✓	✓								✓				✓
Prove results involving divisibility and greatest common divisors						✓	✓		✓								✓		✓	✓	✓
Solve systems of linear congruences							✓		✓								✓		✓	✓	✓

Find integral solutions to specified linear Diophantine equations							✓		✓								✓		✓	✓	✓
Apply Euler-Fermat's Theorem to prove relations involving prime numbers and integers							✓		✓								✓		✓	✓	✓
Apply Wilson's theorem							✓		✓								✓		✓	✓	✓
Demonstrate knowledge of the Legendre symbol and quadratic reciprocity law							✓		✓								✓		✓	✓	✓

C. Course Outline

Time Allotment	Topics
6 hrs	Unit I: Preliminaries • The Number System • Review of Principle of Mathematical Induction and Pigeonhole Principle • Divisibility; division algorithm, GCD, Euclidean algorithm • Bezout's identity; relatively prime; LCM
12 hrs	Unit II: Primes • Fundamental Theorem of Arithmetic • Prime distributions • Fermat and Mersenne primes • Linear Diophantine equations • Primality Testing and Factorization
1.5 hrs	Exam 1
12 hrs	Unit III: Congruences, Roots and Indices • Modular Arithmetic, congruences and congruence classes • Simultaneous congruences and the Chinese Remainder Theorem • Wilson's Theorem and Fermat's Little Theorem • Euler phi-function • Primitive Roots and Indices • Computing powers and roots
1.5 hrs	Exam 2
12 hrs	Unit IV: Quadratic Reciprocity and Other Topics • Quadratic residues • Legendre symbol • Reciprocity laws • Arithmetic functions • Pythagorean triples • Which primes are sums of two powers? • No fourth power is the sum of two fourth powers
	Special Topics* • Elliptic curves and Fermat's Last Theorem • Some unsolved problems in Number Theory
1.5 hrs	Exam 3

*optional

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

References

- K. H. Rosen, Elementary Number Theory and its Applications
- G. Jones and M. Jones, Elementary Number Theory

Further reading

- I. Niven and H. Zuckerman, An Introduction to the Theory of Numbers
- G.H. Hardy and E.M. Wright, An Introduction to the Theory of Numbers
- C. Vanden Eynden, Elementary Number Theory
- K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory
- G.H. Hardy, A Mathematician's Apology
- Lewis Carroll, Alice in Wonderland

FUNDAMENTAL CONCEPTS OF MATHEMATICS

A. Course Details

COURSE NAME	**Fundamental Concepts of Mathematics**
COURSE DESCRIPTION	This course covers sets, principles of logic, methods of proof, relations, functions, integers, binary operations, complex numbers, matrices and matrix operations, and an introduction to mathematical systems.
NUMBER OF UNITS (Lec)	3 units (Lec)
PREREQUISITE	Precalculus

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
determine whether two propositions or predicates are logically equivalent							✓									✓				✓	
construct truth tables							✓									✓					
use and interpret set notation correctly							✓		✓							✓	✓				

construct and understand proofs of mathematical propositions which use some standard proof techniques							✓		✓							✓	✓		✓	✓	✓
determine whether a given relation is an equivalence relation							✓									✓			✓	✓	✓
obtain the equivalence classes that arise from an equivalence relation							✓		✓							✓	✓		✓		
determine whether a given function is injective, surjective or bijective.							✓		✓							✓	✓		✓		
determine whether a given set is finite, countably infinite or uncountable							✓		✓							✓	✓		✓		
determine the cardinality of a given set							✓		✓							✓	✓		✓		

C. Course Outline

Week	Topics
1-2	Sets • Basic definitions and notation • Set operations, algebra of sets • Venn diagrams • Counting properties of finite sets
3-4	Principles of Logic • Statements, logical connectives • Validity, truth table • Tautologies • Quantifiers
5-7	Methods of Proof • Direct proof • Indirect proof • Proof by specialization and division into cases • Mathematical induction
8-9	Relations • Definition • Equivalence relations • Equivalence classes and partitioning • Partial ordering
10-11	Functions • Injection, surjection, bijection • Image, inverse image • Inverse function • Cardinal number of a set • Counting principles • Countable and uncountable sets

12-13	Integers • Divisibility • Division algorithm, Euclidean algorithm • Fundamental Theorem of Arithmetic
14-15	Binary Operations • Definition • Modular operations • Operations on matrices • Operations on complex numbers
16	Introduction to Mathematical Systems • Semigroup • Group • Ring • Field

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Morash. Bridge to Abstract Mathematics
- Gerstein. Introduction to Mathematical Structures and Proofs
- Rotman. Journey to Mathematics
- Kurtz. Foundations of Abstract Mathematics
- Sundstrom. Mathematical Reasoning: Writing and Proofs
- Chartrand, Polimeni and Zhang. Mathematical Proofs: A transition to advanced mathematics

FUNDAMENTALS OF COMPUTING I

A. Course Details

COURSE NAME	**Fundamentals of Computing I**
COURSE DESCRIPTION	This course introduces fundamental programming constructs: types, control structures, functions, I/O, basic data structures using the C programming language. In-class lectures and discussions are supplemented by computer hands-on sessions.
NUMBER OF UNITS	3 units (Lec/Lab)
PREREQUISITE	Precalculus

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	PROGRAM OUTCOME																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
write simple programs in C or other programming language, using the correct syntax, commands, functions, etc.									✓									✓			
design and complete a program to solve a nontrivial mathematical problem.							✓		✓									✓			

C. Course Outline

Week	Topics
1-2	Introduction to Computer Programming • Basic components of a computer • Overview of programming languages • Number systems and conversions • Overview of command shell • Problem-solving on a computer Introduction to C Language • Syntax and semantics • Elements of a C program • Basic I/O: printf, scanf
3-5	Basic DataTypes Identifiers, Keywords, Variables, Constants Operators and Precedence • Arithmetic • Boolean • Relational • Increment/Decrement
6-7	Type Conversions Control Structures • Statements and blocks • Conditional: if-else, switch, ternary operator • Looping: while, do-while, for • Others: break, continue
8-9	Functions Procedures
10-12	Arrays and Strings Pointers User-Defined Data Types
13-14	Manipulating Files Searching and Sorting • Linear search • Binary search • Bubble search

D. Suggested Teaching Strategies

- Lectures, case studies, programming exercises, group discussions, computer demonstrations

E. Suggested Assessment / Evaluation

- Quizzes, midterm exam, final exam, machine problems, programming project

F. Learning Resources

a. Textbooks/References
   - Kernighan and Ritchie. The C Programming Language
   - Kelly and Pohl. C by Dissection-The Essentials of C Programming
   - Goldstein and Gritz. Hands-on Turbo C

FUNDAMENTALS OF COMPUTING II

A. Course Details

COURSE NAME	**Fundamentals of Computing II**
COURSE DESCRIPTION	This course covers advanced programming concepts and techniques using Java, C++ or other suitable object-oriented programming languages. Topics include recursion, abstract data types, advanced path structures, programming interfaces, object-oriented programming, inheritance, polymorphism, event handling, exception handling, API programming. In-class lectures and discussions are supplemented by computer hands-on sessions.
NUMBER OF UNITS	3 units (Lec/Lab)
PREREQUISITE	Fundamentals of Computing I

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
recall the correct usage of the various functions and keywords of the programming language																					
write short programs using the appropriate functions and procedures of the language									✓								✓	✓			
determine if a completed program, or a part of it, runs according to the specified requirements									✓								✓				

design and complete a working program that combines several parts and modules to solve a mathematical problem							✓		✓								✓	✓			
analyze the efficiency of a completed program							✓		✓								✓				

C. Course Outline

Week	Topics
1-2	The way of the program • The Python programming language • What is a program? • What is debugging? • Formal and natural language Variables, expressions and statements • Values and types • Variables • Variable names and keywords • Statements • Operators and operands • Expressions • Order of operations • String operations Functions • Function calls • Type conversion functions • Math functions • Composition • Adding new functions • Definitions and uses • Flow of execution • Parameters and argument
3-5	Conditionals and recursion • Modulus operator • Boolean expressions • Logical operators • Conditional execution • Alternative execution • Chained conditionals • Nested conditionals • Recursion • Stack diagrams for recursive functions • Infinite recursion • Keyboard input Fruitful functions • Return values • Incremental development • Composition • Boolean functions • More recursion

	Iteration • Multiple assignment • Updating variables • The while statement • Break Strings • A string is a sequence • Traversal with a for loop • String slices • Strings are immutable • Searching • Looping and counting • String methods • The in operator • String comparison
6-8	Lists • A list is a sequence • Lists are mutable • Traversing a list • List operations • List slices • List methods • Map, filter and reduce • Deleting elements • Lists and strings • Objects and values • Aliasing • List arguments Dictionaries • Dictionary as a set of counters • Looping and dictionaries • Reverse lookup • Dictionaries and lists • Memos • Global variables Tuples • Tuples are immutable • Tuple assignment • Tuples as return values • Variable-length argument tuples • Lists and tuples • Dictionaries and tuples • Comparing tuples • Sequences of sequence
9-12	Files • Persistence • Reading and writing • Format operator • Filenames and paths • Catching exceptions • Databases • Pickling

	• Pipes • Writing modules Classes and objects • User-defined types • Attributes • Rectangles • Instances as return values • Objects are mutable • Copying Classes and functions • Time • Pure functions • Modifiers • Prototyping versus planning Classes and methods • Object-oriented features • Printing objects • The init method • The str method • Operator overloading Type-based dispatch • Polymorphis • Inheritance • Card objects • Class attributes • Comparing cards • Decks • Printing the deck • Add, remove, shuffle and sort • Inheritance • Class diagrams

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry, programming exercises, computer lab sessions

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Downey, Allen. Think Python. O'Reilly Media. 2012. Also accessible at http://faculty.stedwards.edu/mikek/python/thinkpython.pdf
- Zelle, John. Python Programming: An Introduction to Computer Science, 2nd Edition. Franklin, Beedle and Associates Inc. 2010.
- "The Python Tutorial". Docs.Python.Org. October, 2013. <docs.python.org/3/tutorial/index.html>
- "Non-Programmer's Tutorial for Python 3". Wikibooks. October, 2013 < http://en.wikibooks.org/wiki/Non-Programmer%27s_Tutorial_for_Python_3>

LINEAR ALGEBRA

A. Course Details

COURSE NAME	**Linear Algebra**
COURSE DESCRIPTION	This course covers matrices, systems of linear equations, vector spaces, linear independence, linear transformations, determinants, eigenvalues and eigenvectors, diagonalization, and inner product spaces.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Fundamental Concepts of Mathematics
COREQUISITE	Abstract Algebra I

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
define and illustrate basic concepts in Linear Algebra.						✓							✓			✓					✓
apply the appropriate tools of Linear Algebra to obtain the solutions to a given problem.						✓	✓		✓				✓			✓	✓	✓	✓		
construct a basis for a given vector space or subspace.						✓	✓		✓				✓			✓			✓		
represent linear transformations and quadratic forms with matrices, and describe properties of these functions based on the matrix representation.						✓	✓		✓				✓			✓	✓	✓	✓		
determine the eigenvalues and associated eigenvectors of a matrix/linear transformation							✓		✓		✓		✓			✓		✓			
use the Gram-Schmidt orthonormalization process to construct an orthonormal basis for a given inner product space						✓	✓		✓		✓		✓			✓	✓	✓			

C. Course Outline

Week	Topics
1	• Matrices • Matrix operations and their properties • Transpose of a matrix
2	• Special types of square matrices • The echelon form of a matrix • Elementary matrices and row equivalence
3	• Systems of linear equations • The Inverse of a Matrix
4-5	• Determinants and their properties • Cofactors • The adjoint of a matrix • Cramer's rule
6-8	• Vector Spaces: Definition and examples • Subspaces • Linear Combinations and spanning sets • Linear Independence • Basis and dimension • Rank of a matrix
9-11	• Isomorphism of vector spaces • Linear transformations: definitions and examples • Kernel of a linear transformation • Range, nullity and rank • Dimension Theorem • Nonsingular linear transformations • Matrix of a linear transformation • Similarity
12-13	• Eigenvalues and eigenvectors • Characteristic polynomial • Hamilton-Cayley theorem • Diagonalization
14-15	• Inner product spaces • Orthogonal basis • Gram-Schmidt orthogonalization • Diagonalization of symmetric matrices
16	• Quadratic forms • Positive definite matrices

D. Suggested Teaching Strategies

- Lectures, exercises, group discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, long exams, midterm, final exam, exploration (real-life application of linear algebra)

F. Learning Resources

A. References

- Kolman. Elementary Linear Algebra
- Finkbeiner. Introduction to Matrices and Linear Transformations
- Herstein. Topics in Algebra
- Lang. Linear Algebra

MATHEMATICAL MODELING

A. Course Details

COURSE NAME	**Mathematical Modeling**
COURSE DESCRIPTION	This course is an application of mathematics to various fields. It introduces discrete and continuous models, model fitting and optimization. Applications involve real-world problems from business, engineering, and life sciences. Lectures are supplemented by computer laboratory sessions.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Fundamentals of Computing I, Differential Equations I, Linear Algebra

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
identify a problem involving a physical system						✓	✓	✓			✓		✓	✓		✓	✓	✓		✓	✓
make assumptions on a physical system and develop a mathematical model						✓	✓	✓	✓		✓		✓	✓		✓	✓	✓		✓	✓
apply appropriate mathematical tools to solve the problem						✓	✓	✓	✓		✓			✓		✓	✓	✓			
analyze and validate the model						✓	✓	✓	✓		✓		✓	✓	✓	✓	✓	✓	✓		
rework the model, if necessary						✓	✓	✓	✓		✓		✓			✓	✓	✓		✓	
assess and articulate what type of modeling techniques are appropriate for a given physical system						✓	✓	✓	✓	✓	✓		✓			✓	✓	✓		✓	✓
make predictions of the behavior of a given physical system based on the analysis of its mathematical model						✓	✓	✓	✓	✓	✓				✓	✓	✓	✓		✓	✓

C. Course Outline

Week	Topics
1-4	Discrete Models • Linear models • Discrete models • Systems
5-7	Model Fitting • Fitting model to data graphically • Analytical methods of model fitting • Applying the least-squares criterion • Choosing a best model
8-10	Optimization of Discrete Models • Linear Programming I – geometric solutions • Linear Programming II – algebraic solutions • Linear Programming III – the simplex method
11-16	Continuous Models • Linear models • Nonlinear models • Systems

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, computer sessions, group work, individual inquiry

E. Suggested Assessment / Evaluation

- Problem sets, seatwork, final exam

F. Learning Resources

A. References

- Frank R. Giordano, William P. Fox and Steven B. Horton. A first Course in Mathematical Modeling, 5th Ed., Cengage Learning, 2014.
- Linda S.J. Allen, An Introduction to Mathematical Biology, Pearson-Prentice Hall, 2007..
- Leah Edelstein-Keshet, Mathematical Models in Biology, SIAM (Society for Industrial and Applied Mathematics), Philadelphia (reprint), 2004.
- Cleve Moler, Experiments with MATLAB, MathWorks, Inc., 2001, http: //mathworks.com/moler
- Douglas D. Mooney and Randall J. Swift, A Course in Mathematical Modeling, Mathematical Association of America, 1999.

MODERN GEOMETRY (EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY)

A. Course Details

COURSE NAME	**Modern Geometry (Euclidean and Non-Euclidean Geometry)**
COURSE DESCRIPTION	The first part of the course focuses on Euclidean and affine geometry on the plane. The second half may continue with Euclidean geometry on the sphere; alternatively, an introduction to finite geometries and to the non-Euclidean hyperbolic and elliptic geometries may be given. This course interrelates and makes use of tools from Geometry, Linear Algebra and Abstract Algebra.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Linear Algebra and Abstract Algebra I

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
prove geometric statements using a variety of methods (e.g. synthetic, analytic) with appropriate logical arguments and mathematical rigor.						✓			✓							✓	✓		✓	✓	
identify desirable features of axiomatic and deductive systems such as consistency and completeness.						✓			✓							✓	✓		✓	✓	
describe the basic transformations (e.g. Euclidean, affine, orthogonal)						✓			✓							✓	✓		✓	✓	
explain the significance of Euclid's Fifth Postulate and construct equivalent statements						✓			✓							✓	✓		✓	✓	
describe some models of non-euclidean geometries and finite geometries						✓			✓							✓	✓		✓	✓	
identify properties of small finite geometries						✓			✓							✓	✓		✓	✓	
evaluate the role and contributions of geometry to mathematics, culture and society						✓			✓	✓						✓	✓	✓	✓	✓	

C. Course Outline

Week	Topics
1-5	A. Plane Euclidean Geometry • Review 　○ Coordinate Plane 　○ The Vector Space R^2 　○ The Inner-Product Space R^2 　○ The Euclidean Plane E^2 • Lines • Orthonormal pairs • Equation of a line • Perpendicular lines • Parallel and intersecting lines • Reflections • Congruence and isometries • Symmetry groups • Translations, Rotations, Glide reflections • Structure of the isometry group • *Fixed points and fixed lines of isometries*
6-8	B. Affine Transformations in the Euclidean Plane* • Affine transformations • Fixed lines • The 2-dimensional affine group • Fundamental theorem of affine geometry • Affine reflections • Shears • Dilatations • Similarities • Affine symmetries
9-11	C. Geometry on the Sphere* • Preliminaries from 3-dimensional Euclidean space • The cross-product • Orthogonal bases • Planes • Incidence geometry of the sphere • The triangle inequality • Parametric representation of lines • Perpendicular lines • Motions of the sphere • Orthogonal transformations of Euler's theorem • Isometries • *Fixed points and fixed lines of isometries*
9-11	D. Finite Geometries* • Introduction to finite geometries 　○ Axiomatic systems 　○ Four-line and four point geometries • Finite geometries of Fano and Young • Finite geometries of Pappus and Desargues • Finite geometries as linear spaces 　　a. Near-linear and linear spaces

	b. Incidence matrices c. Numerical properties • Finite projective planes and projective spaces • *Finite affine spaces*
12-16	E. Non-euclidean geometries • Euclid's Fifth Postulate • Introduction to hyperbolic geometry ○ Fundamental postulate of hyperbolic geometry ○ Ideal points and omega triangles ○ Quadrilaterals and triangles • Introduction to elliptic geometry ○ Characteristic postulate of elliptic geometry ○ Quadrilaterals and triangles

Note: *Two of the three main topics in weeks 6 to 11 may be chosen. Italicized items are optional.*

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam, individual/group project

F. Learning Resources

A. References

- Ryan. Euclidean and Non-Euclidean Geometry (for weeks 1, 2 and 3)
- Wald. Geometry: An Introduction
- Greenberg. Euclidean and Non–Euclidean Geometries: Development & History
- Batten, Combinatorics of Finite Geometries (for week 4)
- Smart, Modern Geometries (for week 5)

MODERN GEOMETRY (PROJECTIVE GEOMETRY)

A. Course Details

COURSE NAME	**Modern Geometry (Projective Geometry)**
COURSE DESCRIPTION	This course covers projective planes, projectivities, analytic projective geometry, cross ratio and harmonic sequences, geometric transformations, and isometries.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Linear Algebra

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	PROGRAM OUTCOME																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
show how projective geometry relates to Euclidean geometry									✓							✓	✓		✓	✓	
describe the properties of projective geometry and projective planes									✓							✓	✓		✓	✓	
illustrate the principle of duality									✓							✓	✓		✓	✓	
define the terms "point at infinity (or ideal point)" and "line at infinity."									✓							✓	✓		✓	✓	
outline the proofs and consequences of the theorems of Pappus and Desargues									✓							✓	✓		✓	✓	
illustrate the concepts of "perspective from a line" and "perspective from a point"									✓							✓	✓		✓	✓	
compute the cross ratio and illustrate its projective invariance									✓							✓	✓				
illustrate harmonic sets, harmonic conjugates and complete quadrangles									✓							✓	✓				
work effectively with homogeneous coordinates									✓							✓	✓				

C. Course Outline

Week	Topics
1	Introduction and Historical Background • From Euclidean geometry to non-Euclidean geometry • Some geometries: hyperbolic, elliptic, inversive and projective
2-3	The Projective Plane • Axioms of the projective plane • Principle of duality • Number of points/lines n a finite projective plane • Applications
4-5	Triangles and Quadrangles • Definitions • Desarguesian plane • Harmonic sequence of points/lines
6-7	Projectivities • Central perspectivity • Projectivity • Fundamental theorem of projective geometry • Theorem of Pappus

8-9	Analytic Projective Geometry • Projective plane determined by a three-dimensional vector space over a field • Homogeneous coordinates of points/lines • Line determined by two points • Point determined by two lines • Collinearity, concurrency
10-11	Linear Independence of Points/Lines • Definition • Analytic proof of some theorems like Desargues' Theorem
12	The Real Projective Plane • Ideal points • Ideal line
13	Matrix Representation of Projectivities • Derivation of matrix representation • Fundamental theorem of projective geometry (analytic approach)
14	Geometric Transformations* • Affine transformations and the affine plane • Similarity transformation • Homothetic transformation
15-16	Isometries* • Types of isometries • Products of isometries • Application of isometries to the solution of some geometric problems

*If time permits

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Coxeter and Greitzer. Geometry Revisited
- Smart. Modern Geometry
- Hughes and Piper. Projective Planes

NUMERICAL ANALYSIS

A. Course Details

COURSE NAME	Numerical Analysis
COURSE DESCRIPTION	This is an introductory course that covers error analysis, solutions of linear and nonlinear equations and linear systems, interpolating polynomials, numerical differentiation and integration, numerical approximations of eigenvalues, and numerical solutions of ordinary differential equations.
NUMBER OF UNITS	3 units (Lec/Lab)
PREREQUISITE	Differential Equations I and Linear Algebra

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	PROGRAM OUTCOME																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
choose and use the appropriate method to obtain a numerical solution to a given mathematical problem.						✓	✓		✓				✓				✓				
implement a specified numerical method using available software.							✓		✓									✓			
compute the error of the estimate provided by a given numerical method.																					
compare the accuracy of the estimates provided by different numerical methods for solving a given problem.							✓	✓	✓				✓				✓	✓		✓	✓
discuss a real-life application of a numerical method.							✓	✓	✓		✓		✓				✓	✓			✓

C. Course Outline

Time Allotment	Topics
2 hours	Mathematical Preliminaries • Intermediate Value Theorem • Extreme Value Theorem • Rolle's Theorem and the Mean Value Theorem • Taylor's Theorem

4 hours	Error Analysis and Computer Arithmetic • Floating point arithmetic • Error • Accuracy • Convergence of solutions
7 hours	Solutions of Nonlinear Equations • Bracketing methods • Fixed Point methods • Newton's method • Secant method
6 hours	Solutions of Linear Systems • Gaussian elimination • LU-Decomposition • Gauss-Seidel method • Gauss-Jacobi method
8 hours	Numerical Interpolation • Lagrange Interpolation • Divided differences • Interpolation at equally spaced points: Newton's and Gauss' formulas • Cubic splines
8 hours	Numerical Differentiation and integration • Newton's formulas • Finite differences • Trapezoidal rule • Simpson's rules • Romberg integration • Gaussian integrals

8 hours	Numerical Solutions of Ordinary Differential Equations
	• One-step methods
	    ○ Euler's method
	    ○ Taylor series method
	    ○ Runge-Kutta methods
	• Multi-step methods
	    ○ Adams' corrector-predictor formulas
	    ○ Milne's method

5 hours	Numerical Approximation of Eigenvalues and Eigenvectors • Power method • Inverse power and shifted power method • Rayleigh quotients • QR-Algorithm

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry, computer lab sessions,

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Atkinson. Elementary Numerical Analysis
- Gerald and Wheatley. Applied Numerical Analysis
- Kreysig. Advanced Engineering Mathematics
- Sastry. Introductory Methods of Numerical Analysis
- Scheid. Theory and Problems of Numerical Analysis

OPERATIONS RESEARCH I

A. Course Details

COURSE NAME	**Operations Research I**
COURSE DESCRIPTION	This course is an introduction to linear programming. It covers basic concepts, problem formulation, graphical solution for two-variable problems, simplex algorithm and other algorithms for special LP problems, duality and sensitivity analysis. In-class lectures and discussions are supplemented by computer hands-on sessions.
NUMBER OF UNITS	3 units (Lec/Lab)
PREREQUISITE	Linear Algebra

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
determine appropriateness of linear programming (LP) modeling as framework to investigate real-world problems								✓	✓		✓					✓	✓			✓	
develop LP models that consider key elements of real world problems								✓	✓							✓	✓		✓	✓	
solve the models for their optimal solutions									✓							✓	✓		✓	✓	
interpret the models' solutions and infer solutions to the real-world problems								✓	✓							✓	✓		✓	✓	
illustrate proficiency in the use of the simplex method and its variants and extensions									✓							✓	✓		✓	✓	

apply the principle of duality in solving LP problems									✓							✓	✓		✓	✓	
demonstrate proficiency in using appropriate mathematical software in solving problems.									✓							✓	✓		✓	✓	
apply parametric and integer programming whenever appropriate.								✓	✓							✓	✓		✓	✓	
develop a report that describes the formulation of a model, its solution, and analysis, with recommendations in language understandable to decision-makers		✓				✓		✓	✓							✓	✓		✓	✓	✓

C. Course Outline

Week	Topics

1	Overview of Operations Research
	• Definition of OR
	• The general optimization problem
	    ○ Survey of applications and introduction to some classical LP models
	    ○ The product mix problem
	    ○ The diet problem
	    ○ The transportation problem
	    ○ The fluid bending problem
	    ○ The caterer's problem

2	Linear Programming (LP)
	• Definition of linear programming
	• Formulation of verbal problems into LPs
	• Assumptions/Limitations:
	    ○ Proportionality
	    ○ Additivity
	    ○ Divisibility
	    ○ Nonnegativity
	    ○ Certainty
	    ○ Single objective

3	Geometry of LP in Two Variables • Graphing of linear inequalities • The feasible region as a convex polyhedral area • Geometric interpretation of convex combination • The extreme points • The objective function as a family of parallel lines
4	Review of Linear Algebra • Systems of linear equations • Canonical forms • Basic solutions • Basic feasible solution • Degenerate solutions • Inconsistent systems

	• Pivoting as a sequence of elementary row operations or a sequence of algebraic substitutions
5	Equivalent Formulations of an LP • The use of slacks and surpluses • How to handle variables with no sign restrictions • The symmetric forms • The standard form of an LP • The adjoined form • The canonical forms • The feasible canonical forms • Tableau conventions and notation • Conversion from maximization to minimization
6-7	The Simplex Algorithm • A simple illustration • The Fundamental Theorem of LP and its proof • Details of the algorithm • Possible entrance rules • The exit rule (minimum ratio test) • Test of optimality • Questions of uniqueness • The need for the nondegeneracy assumption
8	The Two-Phase Simplex Method • Artificial variables • Phase I as a test of feasibility • Phase I and algebraic redundancy • The Big M method
9	Revised Simplex Method
10	Duality in LP • The concept of duality • Dual linear programs in symmetric form • Duality theorems • Solving an LP problem from its dual
11	Sensitivity Analysis
12	Parametric Programming
13-14	Integer Programming
15	Special Purpose Algorithm • Transportation problem • Assignment problem • *Maximal flow problem* • *Traveling salesman problem*
16	Computer Applications

Note: Italicized items are optional topics.

D. Suggested Teaching Strategies

- lectures, discussion, individual/group project, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, machine problems, programming exercises, long exams, midterm exam, final exam

and vice versa																					
name some commonly used special discrete and continuous distributions and their properties																					✓
give examples of experiments yielding special distributions																					
compute probabilities, means, and variances of special probability distributions							✓														
derive the distribution of a function of random variables using different techniques							✓											✓		✓	
explain the notion of a random vector																					✓
explain and give the properties of a joint cumulative distribution function and joint probability distribution																					✓
derive conditional distributions and marginal distributions																		✓		✓	
explain and show independence of random variables																		✓		✓	✓
compute mathematical and conditional expectations involving functions of a random vector							✓											✓			
construct sampling distributions and compute their means and variances							✓											✓			
explain the law of large numbers and the Central Limit Theorem																				✓	✓
discuss the importance of the Central Limit Theorem																					✓

C. Course Outline

Week	Topics
1-2	Probability • Sample spaces and events • Methods of assigning probabilities • Axiomatic approach to probability • Calculating probabilities • Conditional probability and independence • Bayes' rule
3-5	Random Variables, Distribution Functions and Expectation • Random variables • Distribution functions- definition and properties • Discrete and continuous random variables • Mathematical Expectation

6-8	Some Special Distributions • Discrete probability distributions-uniform, Bernoulli/binomial, Poisson, hypergeometric, and negative binomial/geometric distributions • Continuous probability distributions: uniform, normal/standard normal, gamma/exponential, Beta, Weibull, Cauchy
9-10	Functions Of Random Variables • Mathematical formulation • Distribution of a function of random variables-CGF technique, MGF technique, method of transformations • Expectation of functions of random variables
11-12	Joint and Marginal Distributions • The notion of a random vector • Joint distribution functions • Marginal distributions • Mathematical expectations
13-14	Conditional Distribution and Stochastic Independence • Conditional distributions • Stochastic independence • Mathematical expectation
15-16 4 hours	Sampling and Sampling Distributions
16-17 4 hours	Laws of Large Numbers and the Central Limit Theorem

D. Suggested Teaching Strategies

- Lecture, discussion, exercises (seatwork, boardwork, assignments, recitation, group work)

E. Suggested Assessment / Evaluation

- Class participation (recitation/ boardwork), Assignment, problem sets, quizzes, final exam

F. Learning Resources

A. References

- Hogg, Craig and Mckean. Introduction to Mathematical Statistics
- Larsen and Marx. Introduction to Mathematical Statistics and Its Applications
- Mood, Graybill and Boes. Introduction to the Theory of Statistics
- Ross. A First Course in Probability

A. Course Details

COURSE NAME	**Real Analysis**
COURSE DESCRIPTION	This course provides an introduction to measure and integration theory. It develops the theory of Lebesgue measure and integration over the real numbers. The course covers topics like the real number system, measurable functions, measurable sets, convergence theorems, integrals of simple and nonnegative measurable functions, and Lebesgue integral.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Advanced Calculus I

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
demonstrate understanding of inner and outer measure by providing examples.									✓							✓	✓			✓	
determine when a set or function is measurable.									✓							✓	✓		✓	✓	
compare Riemann and Lebesgue integration.									✓							✓	✓			✓	
compute and solve Lebesgue integrals									✓							✓	✓			✓	
be familiar with the proof and applications of Fatou's Lemma and other convergence theorems.									✓							✓	✓		✓	✓	

C. Course Outline

Week	Topics
1	Introduction • Comparison between Lebesgue and Riemann integral • Countable and uncountable sets • The extended real number system • Infinite limits of sequences
2	Measurable functions Integral • Measurable sets • Measurable functions
3	Measures • Lebesgue measure • Measure spaces

4	Integrals • Simple functions and their integrals • The integral of a non-negative extended real-valued measurable function • The monotone convergence theorem • Fatou's lemma and properties of integrals
5	Integrable functions • Integrable real-valued functions • The positivity and linearity of the integral • The Lebesgue dominated convergence theorem
6	Modes of convergence • Relations between convergence in mean • Uniform convergence • Almost everywhere convergence • Convergence in measure • Almost uniform convergence • *Egoroff's Theorem* • *Vitali Convergence Theorem*
7	*The Lebesgue spaces Lp* • *Normed linear spaces* • *The Lp spaces* • *Holder's inequality* • *The completeness theorem* • *The Riesz's representation theorem for Lp*

Note: *Italicized items are optional topics.*

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Bartle. Elements of Integration and Lebesque Measure
- Chae and Soo Bong . Lebesgue Integration
- Royden. Real Analysis

A. Course Details

COURSE NAME	**Statistical Theory**
COURSE DESCRIPTION	This course is an introduction to statistics and data analysis. It covers the following: reasons for doing Statistics, collection, summarization and presentation of data, basic concepts in probability, point and interval estimation, and hypothesis testing.
NUMBER OF UNITS	3 units (Lec/Lab)
PREREQUISITE	Fundamental Concepts of Mathematics, Calculus III

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
demonstrate knowledge of the basic terms, concepts and procedures in statistics;						✓															
use appropriate methods of data collection and presentation;						✓	✓		✓								✓	✓			✓
summarize data using different numerical measures						✓	✓		✓		✓				✓		✓	✓			✓
demonstrate knowledge of the basic terms, concepts and procedures in statistics;						✓															
use appropriate methods of data collection and presentation;						✓	✓		✓								✓	✓			✓
apply rules of probability in handling probability sampling distributions;						✓	✓		✓								✓	✓			✓
make inferences about the mean and proportion of one and two populations using sample information through estimation and hypothesis testing;						✓	✓		✓		✓				✓		✓	✓			✓
investigate the linear relationship between two variables by measuring the strength of association and obtaining a regression equation to describe the						✓	✓		✓		✓					✓	✓	✓			✓

relationship																					
analyze data resulting from the conduct of experiments						✓	✓		✓		✓					✓	✓	✓			✓
guard against misuses of statistics						✓										✓	✓				✓

C. Course Outline

Week	Topics
1 (2 hours)	Introduction • Description and history of statistical science • Population and sample
2	Collection and Presentation of Data • Methods of data collection • Probability and non-probability sampling • Tabular and graphical presentations: frequency distribution, stem-and-left display, cross tabulation, histogram
3	Measures of Central Tendency and Location • Arithmetic mean, median and mode • Percentiles
4	Measures of Dispersion and Skewness • Measures of absolute dispersion • Measures of relative dispersion • Measure of skewness • The boxplot
5	Probability • Random experiments, sample spaces, events • Properties of probability
6-7 (5 hours)	Probability Distributions • Concept of a random variable • Discrete and continuous probability distributions • Expected values • The normal distribution • Other common distributions
8	Sampling Distributions
9-10	Estimation • Basic concepts of estimation • Estimating the mean • Estimating the difference of two means *(optional)* • Estimating proportions • Estimating the difference of two proportions *(optional)* • Sample size determination
11-13 (8 hours)	Tests of Hypothesis • Basic concepts of statistical hypothesis testing • Testing a hypothesis on the population mean • Testing a hypothesis on the population proportion • Testing the difference of two means • Testing the difference of two proportions • Test of independence
13-15 (8 hours)	Regression and Correlation • The correlation coefficient

	• Testing a hypothesis on the correlation coefficient • Simple linear regression
16	Analysis of Variance

D. Suggested Teaching Strategies

- Lecture, discussion, exercises, computer laboratory sessions, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, final exam, individual/group reports, problem sets

F. Learning Resources

A. References

- Hayter, A. (2002). Probability and Statistics for Engineers and Scientists (2nd edition). CA: Duxbury.
- Levine, Berenson & Stephan (2002). Statistics for Managers Using Microsoft Excel (3rd edition). Upper Saddle River, NJ: Prentice Hall
- Mann, P. (2010). Introductory Statistics (7th edition). Hoboken, NJ: Wiley.
- Mendenhall, Beaver & Beaver (2009). Introduction to Probability and Statistics (13th edition). Belmont, CA: Thomson/Brooke/Cole.
- Walpole, Myers, Myers & Ye (2005). Probability and Statistics for Engineers and Scientists (7th edition). Singapore: Pearson Education (Asia).

THEORY OF INTEREST

A. Course Details

COURSE NAME	**Theory of Interest**
COURSE DESCRIPTION	This course covers measures of interest, present and future values, equations of value, annuity certains, general annuity certains, yield rates, extinction of debts, and bonds and securities.
NUMBER OF UNITS	3 units (Lec/Lab)
PREREQUISITE	Calculus III

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	PROGRAM OUTCOME																				
At the end of this course, the students should be able to:	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
apply appropriate formulas, concepts and procedures to solve various investment problems.							✓	✓	✓								✓				
distinguish different types of interest rates and how to use these in finding the present value or future value of an investment. Moreover, learn how to compare these rates to make sound judgment as to which rate gives the best return.							✓		✓		✓						✓	✓			
recognize different types of annuities and learn how to find its value at the start, at the end and on any date within or outside its term.							✓		✓		✓						✓	✓			
learn to track the growth/diminution of an investment/a loan.							✓	✓	✓		✓						✓	✓			
determine the value/price, as well as the yield rate of different types of financial instruments like stocks and bonds at different dates during its term.							✓	✓	✓		✓						✓	✓			

C. Course Outline

Time Allotment	Topics
6 hours	Measures of Interest • Accumulation and amount functions • Simple and compound interest • Effective rate of interest • Present and future values • Nominal rates of interest and discount • Force of interest
4 hours	Equations of Value • Present and future values • Current value equation • Unknown time and unknown interest rate
6 hours	Annuity Certain • Annuity immediate • Annuity Due

6 hours	General Annuities • Annuities payable less frequently than interest is convertible • Annuities payable more frequently than interest is convertible • Continuous annuities • Basic varying identities • More general varying identities
8 hours	Yield Rates • Discounted cash flow analysis • Definition of yield rates • Uniqueness of the yield rate • Reinvestment rates • Interest measurement of a fund • Dollar-weighted rate of interest for a single period • Time-weighted rates of interest • Portfolio methods • Investment year methods
6 hours	Extinction of debts • Loan extinction • Computation of the outstanding balance • Amortization method • Sinking fund method
10 hours	Bonds and Securities • Basic financial securities • Bonds and stocks • Price of a bond (FRANK formula) • Other formulas for the bond • Premium and discount • Valuation between coupon payment dates • Yield rates and the Bond Salesman's Formula • Callable bonds • Serial bonds and stocks

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Skills check (boardwork, quizzes, long exam), individual/group report, individual/group project, final exam

F. Learning Resources

A. References

- William Hart. Mathematics of Investment
- Stephen Kellison. The Theory of Interest
- Shao and Shao. Mathematics for Management and Finance

TOPOLOGY

A. Course Details

COURSE NAME	**Topology**
COURSE DESCRIPTION	This course is an introduction to topology. It includes topics fundamental to modern analysis and geometry like topological spaces and continuous functions, connectedness, compactness, countability axioms, and separation axioms.
NUMBER OF UNITS	3 units (Lec)
PREREQUISITE	Advanced Calculus I

B. Course Outcome and Relationship to Program Outcome

COURSE OUTCOMES	**PROGRAM OUTCOME**																				
At the end of this course, the students should be able to:	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**
Determine whether a collection of subsets of a set determines a topology.									✓							✓	✓			✓	
Prove that certain subsets of Euclidean space are topologically equivalent.									✓							✓	✓		✓	✓	
Understand notion of connectedness and be familiar with some standard applications									✓							✓	✓		✓	✓	
Use the definitions of the subspace, product, and quotient topologies to prove their properties and be familiar with standard examples.									✓							✓	✓		✓	✓	
Recognize when a topological space is compact and be familiar with basic properties of compact spaces.									✓							✓	✓		✓	✓	
Develop the concept of metric spaces.									✓							✓	✓		✓	✓	
Recognize when a topological space is connected and be familiar with basic properties of connected sets.									✓							✓	✓		✓	✓	
Demonstrate understanding of countability and separation axioms and illustrate their uses.									✓							✓	✓		✓	✓	

C. Course Outline

Week	Topics
1	Review of Fundamental Concepts of Set Theory and Logic
2	Topological Spaces and Continuous Functions • Topological spaces • Basis for a topology • Continuous functions and homeomorphisms • Construction of subspace, product, quotient, and sum topologies • Closed sets and limit points • The metric topology and the metrization problem
3	Connectedness and Compactness • Connected spaces • Connected sets in the real line • Compact spaces • Tychonoff's Theorem • Compact sets in the real line • Limit point compactness
4	Countability and Separation Axioms • The countability axioms • The separation of axioms and characterization of various spaces • *The Urysohn Lemma: Tietze Extension Theorem* • *The Urysohn Metrization Theorem*

Note: *Italicized items are optional topics.*

D. Suggested Teaching Strategies

- Lectures, exercises, discussion, individual inquiry

E. Suggested Assessment / Evaluation

- Quizzes, problem sets, long exams, midterm exam, final exam

F. Learning Resources

A. References

- Munkres. Topology: A First Course
- Simmons. Topology and Modern Analysis
- Engelking and Sieklucki. Introduction to Topology
- Jänich. Topology
- Kahn. Topology, An Introduction to the Point-Set and Algebraic Areas
- Dixmier. General Topology

ABSTRACT ALGEBRA I **Sample Final Examination**

Directions: Answer the following as indicated and in the given order. In each case, show your complete solution.

I. Answer each item. For true or false items, explain why or give a counterexample if your answer if F. (3 pts each for items 1 to 6)

1. True or False: The element (4,2) of $\mathbb{Z}_{12} \times \mathbb{Z}_8$ has order 12.
2. True or False: The order of the coset $14 + \langle 8 \rangle$ in the factor group $\mathbb{Z}_{24}/\langle 8 \rangle$ is 3.
3. True or False: $12\mathbb{Z}$ is a maximal ideal of $3\mathbb{Z}$.
4. True or False: There is a homomorphism of the symmetric group S_3 into $\mathbb{Z}_6$.
5. Let G and H be groups. What is the kernel of the homomorphism $\varphi : G \times H \to G$ given by the map $(g, h) \mapsto g$.
6. Define: *ideal of a ring* and *prime ideal* and give examples of each.
7. Give an example of a relation on $\mathbb{Z}$ that is not symmetric. (2 pts)

II. Do all. Explain all work. (10 pts each)

1. Let $g = (1\ 4)$, $h = (2\ 1\ 5)$ and $f = (3\ 4\ )$ be permutations in S_5.
   (a) Write ghf as a single permutation.
   (b) Is ghf odd or even?
   (c) What is the inverse of gf?
   (d) What is the order of hf?
2. (a) State Lagrange's Theorem.
   (b) Give one of its corollaries.
   (c) Give a proof of either Lagrange Theorem or the corollary you stated in (b).
3. (a) Solve for x in the equation $x^3+2x^2-3x = 0$ in the ring $(\mathbb{Z}_{11}, +, \cdot)$.
   (b) Find the group of units $(U(\mathbb{Z}_{18}), \cdot)$ in the ring $\mathbb{Z}_{18}$ under addition and multiplication modulo 10.
   (c) What group is $(U(\mathbb{Z}_{18}), \cdot)$isomorphic to?
4. Let $G = \mathbb{Z}_4 \times \mathbb{Z}_4$, under component-wise addition modulo 4. Let $H = \langle (1, 2) \rangle$.
   (a) List down the elements of H and G/H.
   (b) Is $(2, 1) + H = (1, 3) + H$? Why or why not?
   (c) To which known group is G/H isomorphic and why?
5. Let $\phi : G \to G'$ be a homomorphism, where G and G' are finite groups.
   (a) Show that the kernel of ϕ is a subgroup of G.
   (b) Show that $|\phi(G)|$ divides both $|G'|$ and $|G|$.
6. Let G be a group with identity e for which $x^2 = e$ for all x in G.
   (a) Show that G is abelian.
   (b) Show that for any elements $a, b, c, d \in G$, if $ab = cd$ then $ac = bd$.
7. (a) Let $\phi : \mathbb{Z}_8 \to \mathbb{Z}_{24}$ be the homomorphism satisfying $\phi(7) = 4$. Find the kernel of ϕ, and the images of all the elements of $\mathbb{Z}_8$.
   (b) State the 1st Isomorphism Theorem for Groups.

End of Exam (Total: 100 points)

Directions: Answer the following as indicated and in the given order. In each case, show your complete solution.

I. True or False. If false, explain briefly why or give a counterexample. [12 points]

1. $\mathbb{Q}(i, \sqrt{3}) = \mathbb{Q}(i\sqrt{3})$.
2. $\mathbb{Q}[x]/\langle 2x^2 - 7x + 3 \rangle$ is a field.
3. $\mathbb{Z}$ is a unique factorization domain but not a principal ideal domain.
4. The splitting field over $\mathbb{Q}$ of $x^2 - 6x + 7$ is $\mathbb{Q}(\sqrt{2})$.
5. $\sqrt[3]{1 - \sqrt{2}}$ is an algebraic number.
6. The transcendental numbers form a subfield of the real numbers.
7. -1 is a primitive complex 4^{th} root of unity.
8. $[K : \mathbb{Z}_5] = 3$, where K is the splitting field of $x^3 - 3$ over $\mathbb{Z}_5$.

II. Find the following. No need to justify. [8 points]

1. the units in $\mathbb{Z}[i]$
2. a basis of $\mathbb{Q}(1 + \sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$
3. the associates of the element 2 in the ring $\mathbb{Z}_6$
4. the definition of a multiplicative norm on an integral domain
5. the number of elements of $\mathbb{Z}_3[x]/\langle x^3 + 2x + 1 \rangle$
6. the irreducible polynomial of $\sqrt{1 - \sqrt{2}}$ over $\mathbb{Q}(\sqrt{2})$
7. the definition of a separable extension E over a field F
8. the elements of the Galois group of $\mathbb{Q}(\sqrt[3]{2})$ over $\mathbb{Q}$.

III. Justify all work. [28 pts total]

1. Let α and β be non-zero non-unit elements in an integral domain. Show that if $\langle \alpha \rangle = \langle \beta \rangle$ then α and β are associates. [4 pts]
2. Let K be an extension of F with $[K : F] = m$. Show that if $\alpha \in K$ has degree n over F, then $n|m$. [4 pts]
3. Derive the 9^{th} cyclotomic polynomial and write it as a polynomial in $Z[x]$. [4 pts]
4. Let α be a zero of $x^2 + 1$ in some extension field of $\mathbb{Z}_3$. [5 pts]
   (a) List the elements of $\mathbb{Z}_3(\alpha)$ in additive form.
   (b) Is α a generator?
   (c) What is the inverse of $1 + 2\alpha$?
5. Let $K = \mathbb{Q}(i, \sqrt[4]{2})$ over $F = \mathbb{Q}(i)$. [5 pts]
   (a) Find $[K : F]$ and $|G(K/F)|$.
   (b) Find all elements of the group $G(K/F)$. What group is this isomorphic to?
6. Let $\omega = cos\frac{2\pi}{5} + isin\frac{2\pi}{5}$. Let ϕ be the automorphism of $\mathbb{Q}(\omega)$ which sends $\omega \mapsto \omega^2$. [6 pts]
   (a) What is the order of ϕ in the group $G = G(\mathbb{Q}(\omega)/\mathbb{Q})$?
   (b) List the elements of G.
   (c) Determine the fixed subfields of G.

End of Exam

Total points = 50

Please fold and insert this sheet in back of exam booklet.

ADVANCED CALCULUS I **Sample Final Examination**

Directions: Answer the following as indicated and in the given order. In each case, show your complete solution.

1. Let $A \subseteq R$. Define the following: (6 points)
   (a) A is open in R
   (b) $\lim_{n\to\infty} x_n = x$
   (c) $\lim_{x\to a} f(x) = \infty$
   (d) a function f continuous at $a \in A$
   (e) a function f uniformly continuous on a set A
   (f) a sequence of functions f_n converging uniformly to a function f

2. State six (6) the following: (6 points)
   (a) Monotone Convergence Theorem
   (b) Cauchy Convergence Criterion
   (c) Intermediate Value Theorem
   (d) Mean Value Theorem
   (e) Riemann's Criterion for Integrability
   (f) Fundamental Theorem of Calculus (either form)
   (g) Mean Value Theorem for Integrals

3. Prove three of the theorems in (2).(12 pts)

4. Modified True or False. If the statement is true, give a short proof. If false, give a counterexample, explain briefly, and modify the statement to make it true. Choose 12. (36 pts)
   (a) The intersection of a finite collection of open sets is open.
   (b) Every bounded sequence of real numbers has a convergent subsequence.
   (c) Every convergent sequence of real numbers is a Cauchy sequence.
   (d) If $\lim_{x\to a} f(x) > 0$, then there exists a neighborhood V_a of a such that $f(x) > 0$ for every $x \in V_a$, $x \neq a$.
   (e) Let f be differentiable on an interval I. If $f(x)$ is strictly decreasing on I, then $f'(x) < 0$ for all $x \in I$.
   (f) $x < \ln x < x - 1$ for all $x > 1$.
   (g) $\lim_{x\to\infty}[1 + \frac{2}{x}]^x = e$.
   (h) If $f(x)$ satisfies a Lipschitz condition on $A \subset R$, then f is uniformly continuous on A.
   (i) If f and g are uniformly continuous and bounded on A, then their product fg is also uniformly continuous.
   (j) If f is differentiable at $a \in A$, then f is continuous at $a \in A$.
   (k) Let P and Q be partitions of $[a, b]$. If P is a refinement of Q then $L(f, Q) < L(f, P)$.
   (l) Let f be integrable on I. Then there exists $c \in I$ such that $\int_a^b f = f(c)(b - a)$.
   (m) If $F = \int_a^x f$, $x \in [a, b]$, then $F'(x) = f(x)$.
   (n) If f_n converges uniformly to f on A and f_n is continuous on A for every n, then f is continuous on A.
   (o) $\dfrac{nx}{2 + nx}$ converges to 1 uniformly on $[0, \infty)$

END OF EXAM
(60 points)

Directions: Answer the following as indicated and in the given order. In each case, show your complete solution.

1. Show that $(1+i)^{95} = 2^{47}(1-i)$.

2. Let $S \subset \mathbb{C}$. If S is finite, show that S is closed.

3. Let $D = \{\ z \in \mathbb{C}\ :\ -1 < \text{ Im } z \leq 1\ \}$.

   (a) Sketch the given set D.

   (b) Find the interior, exterior and boundary of D.

   (c) Is set D open? closed? connected? Explain each of your answers.

   (d) If $f(z) = iz + 2i$, sketch the image $f(D)$ of D under f.

4. Let $f(z) = \dfrac{x^2}{x^2+y^2} + 2i$. Does $\lim\limits_{z\to 0} f(z)$ exist? Why or why not?

5. (a) Let $f = u + iv$ be analytic in a domain D. If Im $f = v$ is constant, show that f is also constant.

   (b) Use (a) to show that the function defined by $f(z) = \sin xy + \pi i$ is nowhere analytic.

6. Show that the function defined by $f(z) = x^3 + 3xy^2 - 3x + i(y^3 + 3x^2y - 3y)$ is nowhere analytic.

7. Let $u = \cos x \sinh y$.

   (a) Show that u is harmonic in the entire complex plane.

   (b) Find a harmonic conjugate of u.

8. Let u, v be real-valued functions defined in some domain D. If v is a harmonic conjugate of u in D, show that $w = uv$ is harmonic in D.

9. Let $D = \{\ z = x + iy \in \mathbb{C}\ :\ -1 \leq x \leq 1,\ 0 \leq y \leq \pi\ \}$. If $f(z) = e^z$, sketch D and $f(D)$ on different copies of the complex plane.

10. Below is an outline of an alternative proof that $\sin^2 z + \cos^2 z = 1$ for all $z \in \mathbb{C}$. Justify each step of the proof.

    (a) The function $f(z) = \sin^2 z + \cos^2 z$ is entire.

    (b) $f'(z) = 0$ for all $z \in \mathbb{C}$.

    (c) f is a constant function in $\mathbb{C}$.

    (d) $f(0) = 1$.

    (e) $f(z) = \sin^2 z + \cos^2 z = 1$ for all $z \in \mathbb{C}$.

11. Find a Laurent series for the function $f(z) = \dfrac{e^z}{(z+1)^3}$ which is valid in the annular domain $1 < |z+1| < \infty$.

12. In each case, determine whether the singular point is a pole, a removable singular point or an essential singular point:

    (a) $f(z) = z^3 \exp\left(\dfrac{1}{z}\right)$, $z_0 = 0$

    (b) $f(z) = \dfrac{1 - \cosh z}{z^2}$, $z_0 = 0$

    (c) $f(z) = \dfrac{2z}{z(z+1)^2}$, $z_0 = -1$

13. Evaluate each of the following integrals. Cite any theorem which is applicable.

    (a) $\displaystyle\int_C \frac{dz}{z}$, $C:\ z = 2e^{it},\ t \in [-\pi/2,\ \pi/2]$

    (b) $\displaystyle\int_C \coth z\, dz$, $C:\ |z| = 3$

    (c) $\displaystyle\int_C \frac{e^{2z}}{(z^2+16)(z+1)^3}$, $C:\ |z+2| = 3$

    (d) $\displaystyle\int_0^\infty \frac{x^2\, dx}{(x^2+9)^2}$

    (e) $\displaystyle\int_{-\infty}^\infty \frac{\cos 3x}{(x^2+1)^2}\, dx$

14. Let D be the region inside the curve $\{\ z \in \mathbb{C}\ :\ |z - 3i| + |z + 3i| = 10\}$ and outside the curve $\{\ z \in \mathbb{C}\ :\ |z| = 8\ \}$.

    (a) Sketch the region.

    (b) Let $f(z)$ be a continuous function with zeros at the origin and outside the first curve. If $\dfrac{1}{f(z)}$ has the Laurent series representation

    $$\frac{1}{f(z)} = \frac{10}{z^3} - \frac{25}{z} + 3z - 2z^2 + \cdots$$

    evaluate $\displaystyle\int_C \frac{1}{f(z)}\, dz$ where C is any simple closed contour that lies completely inside D.

15. Use Rouche's theorem to show that if $0 < ae < 1$, then the equation $ae^z - z = 0$ has exactly one root interior to the unit circle.

16. Prove any three of the following:

(a) Let f be entire, and let $u(x, y) = \text{Re } f(z)$. If there exists $u_0 \in \mathbb{R}$ such that $u(x, y) \leq u_0$ for all $(x, y) \in \mathbb{C}$, use the function $e^{f(z}$ to show that $u(x, y)$ is constant in $\mathbb{C}$.

(b) Suppose g is analytic and has a zero of order n at z_0. Show that the function f given by

$$f(z) = \frac{g'(z)}{g(z)}$$

has a simple pole at z_0, and $\text{Res}\,(f(z), z_0) = n$.

(c) Let q be analytic at z_0, $q(z_0) = 0$, $q'(z_0) \neq 0$, and define the function $f(z) = \dfrac{1}{(q(z))^2}$.

(i) Show that z_0 is a pole of order 2 of the function f.

(ii) Show that $\text{Res}(f,\ z_0) = -\dfrac{q''(z_0)}{[q'(z_0)]^3}$.

(d) The *Legendre polynomial* $P_n(z)$ is defined by

$$P_n(z) = \frac{1}{2^n n!} \frac{d^n}{dz^n} \left[(z^2 - 1)^n\right]$$

Show that

$$P_n(z) = \frac{1}{2\pi i} \int_C \frac{(s^2 - 1)^n}{2^n s - z)^{n+1}}\, ds$$

where C is a positively oriented simple closed contour whose interior contains z.

DIFFERENTIAL EQUATIONS **Sample Final Examination**

Directions: Answer the following as indicated and in the given order. In each case, show the complete solution and box your final answer.

1. Find the general solution, unless specified otherwise, of each of the following first order equations.

   (a) $x \cos^2 y \; dx + \tan y \; dy = 0$

   (b) $y \; dx = (x + \sqrt{y^2 - x^2}) \; dy$

   (c) $2xy \; dx + (x^2 + y^2) \; dy = 0$, satisfying $y(-1) = 1$

   (d) $(4xy + y^2) \; dx + (2y - 2x^2) \; dy = 0$

2. Find the general solution of each of the following equations:

   (a) $(D^4 + 2D^3 + 10D^2)y = 0$ (b) $(4D^4 - 16D^3 + 7D^2 + 4D - 2)y = 0$

3. Solve the given nonhomogeneous equations using the indicated methods:

   (a) $y^{(3)} - 4y'' + 4y' = 5x^2 - 6x + 4x^2e^{2x} + 3e^{5x}$ (undetermined coefficients)

   (b) $y^{(3)} - 3y'' + 3y' - y = x - 4e^x$ (solution by inspection)

   (c) $y'' - 3y' + 2y = 12xe^{2x} - 4e^x$ (exponential shift)

4. Show that one solution of the equation $(1 - x^2)y'' - 2xy' + 2y = 0$ is the function $y_1 = x$. Use reduction of order to find the general solution.

5. Use variation of parameters to find a particular solution for the equation $(D^2 + 1)y = \sec^2 x \; \csc x$ and use this to find the general solution.

6. Use Laplace and inverse transforms to find the particular solution of the differential equation $(D^2 + 4D + 5)y = 10e^{-3t}$ which satisfies the conditions $y(0) = 4, \; y'(0) = 0$.

7. Find the general solution of each of the following linear systems:

   (a) $\dfrac{dx}{dt} = 3x3ye^{-t} \qquad \dfrac{dy}{dt} = -x - y + e^{2t}$

   (b) $X' = AX$ where $A = \begin{pmatrix} 0 & -1 & 3 \\ 2 & -3 & 3 \\ 2 & -1 & 1 \end{pmatrix}$.

Directions: Answer the following as indicated and in the given order. In each case, show your complete solution.

1. (15 pts) True or False: You don't have to justify your answers.
   (a) $p \to q \equiv \neg(p \wedge \neg q)$
   (b) $(p \to q) \wedge (\neg q) \equiv \neg p$
   (c) $\neg\forall x \exists y\, (P(x,y) \to \neg Q(x,y)) \equiv \exists x \forall y\, (P(x,y) \wedge Q(x,y))$.

2. (15 pts) Prove using a sequence of logical equivalences that $p \to (q \to r)$ and $q \to (p \to r)$ are equivalent.

3. (15 pts) Show that the following argument is valid.

$$\left\{\begin{array}{l} \neg a \vee b \\ \neg(c \wedge \neg a) \\ \neg b \\ d \to (c \vee e) \\ \underline{e \to d} \\ \therefore d \leftrightarrow e \end{array}\right.$$

4. (15 pts) Find the number of ternary strings of length six that have
   (a) exactly two 1's.
   (b) at least two 1's.

5. (20 pts) Find the number of integer solutions to the equation

$$x + y + z + w = 18$$

   (a) if x, y, z and w are nonnegative?
   (b) if $x \geq 2$, $y \geq 3$, $z \geq 4$ and $w \geq 5$?
   (c) if $2 \leq x \leq 4$, $1 \leq y \leq 3$, $3 \leq z \leq 5$ and $4 \leq w \leq 6$, using generating functions.

6. (15 pts) Let $S = \{1, 2, \ldots, 4k+2\}$ for some positive integer k. How many subsets A of S
   (a) have exactly 3 elements?
   (b) have the property that the sum of all elements (in A) is odd? Simplify your answer.

7. (10 pts) Prove by a combinatorial argument

$$\binom{n}{m}\binom{m}{r}\binom{r}{s} = \binom{n}{s}\binom{n-s}{r-s}\binom{n-r}{m-r}$$

   for any nonnegative integers m, n, r, and s with $s \leq r \leq m \leq n$.

8. (15 pts) Show that the sum of a rational number and an irrational number is irrational.

9. (10 pts) Let A and B be sets. Show that $A \cup (B - A) = A \cup B$.

(continued on the next page)

10. (15 pts) Let f be a function from $\mathbb{R}$ to the open interval $(-1, 1)$, with $f(x) = \dfrac{e^x - 1}{e^x + 1}$ for each $x \in \mathbb{R}$. Show that f is a bijection.

11. (20 pts) Find the generating function of the sequence which is a solution to the recurrence relation
$$a_n = 2a_{n-1} + 3a_{n-2} + 2^n,$$
where $a_0 = 1$ and $a_1 = 5$. You don't have to give a closed form for a_n.

12. (20 pts) Solve the recurrence relation using the characteristic equation:
$$\begin{cases} a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}, \text{ if } n \geq 3 \\ \text{where } a_0 = 1,\ a_1 = 6 \text{ and } a_2 = 12. \end{cases}$$

13. (15 pts) Let a_n be the number of binary strings of length n that contain the substring 00 or 11. Find a recurrence relation for $\{a_n\}$ and determine the values of a_1, a_2 and a_3.

END OF EXAM

Directions: Answer the following as indicated and in the listed order. Justify all answers and show all solutions. Proofs should be clear, coherent, complete and concise. Examples should be verified *to satisfy prescribed properties. You may use any result proved in class,* **unless** *what you are being asked to prove is the result itself.*

1. Let $f : A \to B$ be a function and let $r, s \in \mathbb{R}$. Express each sentence using predicates, quantifiers, logical connectives and the symbols $=$ or $<$ such that no negation appears before a quantifier or a logical connective.
   (a) (2 pts.) f is not onto.
   (b) (2 pts.) Every real number that is less than r is less than s.
2. (3 pts.) A student is asked to prove a statement of the form $(p \to r) \vee (p \to q)$. The student begins by assuming p and proving $q \vee r$. Is this approach correct and why?
3. (3 pts.) Let $n \in \mathbb{Z}$. Prove that when $n^2 - n$ is divided by 3, the remainder is 0 or 2.
4. (3 pts.) Use the Principle of Mathematical Induction to prove that for all $n \in \mathbb{N}$, 6 divides $n^3 - n$.
5. (3 pts.) Show that if $a, b, c, d \in \mathbb{Q}$ such that $a + b\sqrt{2} = c + d\sqrt{2}$, then $a = c$ and $b = d$.
6. (3 pts.) *State* the definition of a symmetric relation R on a set A.

   Using definitions only, *prove* that if R is a relation on a set A, then $R \cup R^{-1}$ is a symmetric relation on A.
7. (a) (2 pts.) *Give* an example of an equivalence relation R on $A = \{a, b, c, d\}$ that gives rise to a partition of A into two cells.
   (b) (2 pts.) *How many* distinct equivalence relations are there on $A = \{a, b, c, d\}$ that give rise to a partition of A into two cells and why?
8. (3 pts.) Let A, B and C be sets. *State* the definition of $A \sim B$, that is, A is numerically equivalent to B.

   Prove or *disprove* the following statement by giving an appropriate proof or counterexample:

   $$\text{If } A \times B \sim A \times C, \text{ then } B \sim C.$$
9. (2 pts.) Let A, B and C be sets. *State* the definition of $A \prec B$.

   Prove that

   $$\text{if } A \prec B \text{ and } B \prec C, \text{ then } A \prec C.$$
10. (2 pts.) Give a partition of $\mathbb{R}$ into four cells C_1, C_2, C_3, C_4 such that

    $$C_1 \sim \mathbb{R}, \quad C_2 \sim C_3 \sim \mathbb{N} \quad \text{and} \quad C_4 \sim I_4.$$
11. (2 pts.) Show that there does not exist a set A such that its power set $\mathcal{P}(A)$ is countably infinite.
12. (3 pts.) Let $\mathbb{Q}(\sqrt{2}) \equiv \{\, a + b\sqrt{2} \,|\, a, b \in \mathbb{Q} \,\}$. Show that $\mathbb{Q}(\sqrt{2}) \sim \mathbb{N}$.

END OF EXAM
Total: 35 points

LINEAR ALGEBRA **Sample Final Examination**

Directions: Answer the following as indicated and in the given order.

A. **MULTIPLE CHOICE:** Write the letter that corresponds to the correct answer. If the answer is not among the given choices, write D. *(2 points each)*

1. If A is a 2×3 matrix and B is a 3×2 matrix, then AB is

   A. 3×2 B. 3×3 C. 2×2

2. A square matrix A is *idempotent* if $A^2 = A$. Which of the following matrices is idempotent?

   A. θ_n C. both (A) and (B)
   B. I_n

3. If A and B are both nonsingular $n \times n$ matrices, which of the following is also nonsingular?

   A. $A + B$ B. BA C. $(A - B)^T$

4. Which of the following permutations is even?

   A. 52134 B. 12354 C. 41253

5. Which of the following matrices is singular?

   A. $\begin{pmatrix} 2 & -3 \\ 2 & -4 \end{pmatrix}$ C. $\begin{pmatrix} 2 & -1 \\ 3 & -5 \end{pmatrix}$
   B. $\begin{pmatrix} -3 & -2 \\ 6 & 4 \end{pmatrix}$

6. Let A be a square matrix such that $A^{-1} = A$. The statement "$|A| = 1$" is

   A. always true C. never true
   B. sometimes true

7. If A is a 3×3 matrix whose determinant is -1, then $|-2A^{-1}|$ is equal to

   A. 8 B. $\frac{1}{8}$ C. -8

8. If $A = \begin{pmatrix} 2 & -1 \\ 3 & -5 \end{pmatrix}$, then A_{21} is equal to

   A. 3 B. -3 C. -1

9. Which of the following polynomials belongs to the vector space $W = \{\ f(t) \in P_3 \mid f(1) = 0\ \}$?

   A. $t^3 + 1$ B. $t^2 + t - 2$ C. $t^2 - 2t - 3$

10. Which of the following augmented matrices represents a linear system with a unique solution?

    A. $\left[\begin{array}{rr:r} 1 & -1 & -2 \\ 2 & -3 & 1 \end{array}\right]$ C. $\left[\begin{array}{rr:r} 1 & -1 & -2 \\ 2 & -2 & -4 \end{array}\right]$
    B. $\left[\begin{array}{rr:r} 1 & -1 & -2 \\ -2 & 2 & -1 \end{array}\right]$

11. Which of the augmented matrices in the preceding problem represents a linear system that has infinitely many solutions?

    A. $\left[\begin{array}{rr:r} 1 & -1 & -2 \\ 2 & -3 & 1 \end{array}\right]$ C. $\left[\begin{array}{rr:r} 1 & -1 & -2 \\ 2 & -2 & -4 \end{array}\right]$
    B. $\left[\begin{array}{rr:r} 1 & -1 & -2 \\ -2 & 2 & -1 \end{array}\right]$

12. Which of the following is among the infinitely many solutions of the system described in the preceding question?

    A. $x = 1, y = -1$ C. $x = 3, y = 1$
    B. $x = 0,\ y = 2$

13. Which of the following statements is always true?

    A. Every scalar matrix is diagonal.
    B. Every diagonal matrix is upper triangular.
    C. Both (A) and (B)

14. If $A = \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}$, then A^{-1} is given by

    A. $\begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$ C. $\begin{pmatrix} -2 & -3 \\ 1 & 1 \end{pmatrix}$
    B. $\begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$

15. If $n > 1$ and J_n is the $n \times n$ matrix whose entries are all ones, then $|J_n|$ is equal to

    A. 0 B. 1 C. n

16. Which of the following is a basis for P_2?

    A. $\{\ t, t - 1\}$
    B. $\{\ t^2 + 1, 2t - 1, t + 2, -2\ \}$
    C. $\{t^2,\ t + 1,\ t - 2\ \}$

17. Which of the following mappings from $\mathbb{R}^2$ to $\mathbb{R}$ is a linear transformation?

    A. $L((x,y)) = x^2 + y^2$ C. $L((x,y)) = xy + 1$
    B. $L((x,y)) = 2x - 3y$

For items 18-20, consider the linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $L((x,y,z)) = (x - 2y, x + y - z)$.

18. The dimension of the range of L is

    A. 1 B. 2 C. 3

19. Which of the following belongs to the kernel of L?

    A. $(2,-1,3)$ C. $(-2,1,-3)$
    B. $(-2,-1,-3)$

20. Which of the following is true about the linear transformation L?

    A. L is one-to-one C. Both (A) and (B)
    B. L is onto

For items 21-22, consider a linear transformation $L : \mathbb{R}^2 \to \mathbb{R}$ for which we have $L((1,0)) = 1$ and $L((0,1)) = -1$.

21. Find the image of $X = (1,-2)$.

    A. 3 B. -1 C. 0

22. L is defined by the equation

    A. $L((x,y)) = x + y$ C. $L((x,y)) = -x - y$
    B. $L((x,y)) = -x + y$

For items 23-24 , consider the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$$

23. The characteristic polynomial of A is

    A. $\lambda^2 - 3\lambda + 2$ C. $\lambda^2 - 3\lambda - 4$
    B. $\lambda^2 - 3\lambda + 8$

24. Which of the following is an eigenvalue of the matrix A?

    A. -1 B. -4 C. 1

25. Which of the following vectors is an eigenvector of the matrix $A = \begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$?

    A. $(4\ \ 3)^T$ C. $(2\ \ 1)^T$
    B. $(-1\ \ 0)^T$

26. If the coordinate vector of $X \in \mathbb{R}^2$ with respect to the basis $S = \{\ (1,-1),(2,1)\ \}$ is $[X]_S = [\,2\ \ -1]^T$, then X is equal to

    (A) $(0,-3)$ (C) $(2,-1)$
    (B) $(0,-1)$ (D) $(3,1)$

27. The quadratic form $Q : \mathbb{R}^2 \to \mathbb{R}$ defined by $Q((x,y)) = x^2 + 4y^2$ is

    A. positive definite
    B. negative definite
    C. indefinite

B. **Computations:**

1. let $V = \mathbb{R}^4$ and let $W = \{\ (a,b,c,d) \in V \mid c = a + 2b,\ d = a - 3b\ \}$.
    (a) Show that W is a subspace of V. *(5 points)*
    (b) Find the dimension of W. *(5 points)*
2. Let $L : P_2 \to P_1$ be the linear transformation defined by
    $$L(at^2 + bt + c) = (a+b)t + (b-c)$$
    (a) Find $L(t^2 - 3t + 2)$. *(3 points)*
    (b) Find ker L and use it to determine if L is one-to-one. *(6 points)*
    (c) Find a basis for range L and use this to determine if L is onto. *(6 points)*
3. Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.
    (a) Show that A is diagonalizable by finding a basis S of eigenvectors for $\mathbb{R}^2$. *(10 points)*
    (b) Find a diagonal matrix D which is similar to A. *(3 points)*
4. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -6 \end{bmatrix}$. *(5 points)*
5. Let $A = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$.
    (a) Construct the quadratic form Q defined by

A. *(3 points)*

(b) Find a quadratic form Q' equivalent to Q whose matrix is a diagonal matrix. *(6 points)*

(c) Is Q positive definite? Why or why not? *(3 points)*

6. Use the Gram-Schmidt process to find an orthonormal basis for $V = \mathbb{R}^3$ using the basis $S = \{ (1,1,0), (1,0,1), (0,1,1) \}$.

C. **Proving:** Prove the following statements. *(5 points each)*

1. If A and B are $n \times n$ matrices such that AB is singular, then either A or B is singular.
2. Let $L : V \to W$ be a linear transformation, and let $\dim V = \dim W$. If S is a basis for V, show that L is both one-to-one and onto if $L(S)$ is a basis for W.
3. Let $L : V \to W$ be a linear transformation, and let $S = \{ X_1, X_2, \ldots, X_n \}$ be a subset of V. If $T = \{ L(X_1), L(X_2), \ldots, L(X_n) \}$ is linearly independent in W, show that S is also linearly independent in V.

Directions: Answer the following as indicated and in the given order. In each case, show your complete solution.

1. Let $\mathcal{S}$ be the space of vectors (a, b, c, d) such that $a+c+d = 0$. Let $\mathcal{T}$ be the space of vectors (a, b, c, d) such that $a + b = 0$ and $c = 2d$.

   (a) Find a basis for $\mathcal{S}$. **[10]**

   (b) Find a basis for $\mathcal{T}$. **[10]**

   (c) Find a basis for $\mathcal{S} \cap \mathcal{T}$. **[15]**

2. Evaluate **[15]**

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ r & 1 & 1 & 1 \\ r & r & 1 & 1 \\ r & r & r & 1 \end{vmatrix}.$$

3. If $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ form a basis for a vector space $\mathcal{V}$, show that $\mathbf{x}_1$, $\mathbf{x}_1 + \mathbf{x}_2, \ldots, \mathbf{x}_1 + \mathbf{x}_2 + \cdots + \mathbf{x}_n$ also form a basis for $\mathcal{V}$. **[25]**

4. Let $\mathcal{W}$ be the intersection of the two planes π_1 and π_2 in $\mathbb{R}^3$ defined by **[20]**

$$\pi_1 : x - y - z = 0$$
$$\pi_2 : x + 2y + z = 0.$$

   Find a basis for $\mathcal{W}^{\perp}$.

5. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in an inner product space $\mathcal{V}$. Prove that $\|\mathbf{u}+\mathbf{v}\| = \|\mathbf{u}-\mathbf{v}\|$ if and only if $\mathbf{u} \perp \mathbf{v}$. **[15]**

6. Let $L : \mathbb{R}^{n\times n} \mapsto \mathbb{R}^{n\times n}$ be defined by $L(A) = A - A^T$. Find the dimension of $\ker L$. **[15]**

7. Let : $P_3 \mapsto \mathbb{R}$ be defined by

$$L(p) = \int_0^2 p(t)\, dt.$$

   (a) Find the matrix representation of L with respect to the basis $B_1 = \{t^3, t^2, t, 1\}$ for P_3 and $B_2 = \{1\}$ for $\mathbb{R}$. **[10]**

   (b) Using the matrix in part a, find **[10]**

$$\int_0^2 \left(4t^3 + 3t^2 + 2t + 7\right)$$

   dt

   by matrix multiplication.

   (c) Evaluate the integral in part b directly and compare your answer with the one in part b. **[5]**

8. Let $A \in \mathbb{R}^{n\times n}$ be diagonalizable with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. Show that A can be written as **[25]**

$$A = \lambda_1 P_1 + \lambda_2 P_2 + \cdots +$$

   $\lambda_n P_n$

   where each $P_i \in \mathbb{R}^{n\times n}$ has rank 1.

9. Let $A \in \mathbb{R}^{n\times n}$ be an orthogonal matrix. If $NS\left(A + A^T\right) \neq \{\mathbf{0}\}$, show that $\lambda = -1$ is an eigenvalue of A^2. **[25]**

END OF EXAM

MODERN GEOMETRY — Sample Final Examination

Choose the letter that corresponds to the correct answer. (2 POINTS EACH)

1. Which of the following is an orthonormal basis for $\mathbb{R}^3$?

   A. $\{u, v, u \times v/|u \times v|\}$ with $|u| = |v| = 1$, $\langle u, v\rangle = 0$

   B. $\{u, v, u \times v/|u \times v|\}$ with $|u| = |v| = 0$, $\langle u, v\rangle = 1$

   C. $\{u, v, u \cdot v/|u \cdot v|\}$ with $|u| = |v| = 1$, $\langle u, v\rangle = 0$

   D. $\{u, v, u \cdot v/|u \cdot v|\}$ with $|u| = |v| = 0$, $\langle u, v\rangle = 1$

2. Which of the following in NOT on the line ℓ in S^2 with pole $\psi = (-1, 0, 0)$?

   A. $(0, 0, 1)$ C. $(-1/\sqrt{2}, 1/\sqrt{2}, 0)$

   B. $(0, 1/2, -\sqrt{3}/2)$ D. $(0, 2/\sqrt{5}, 1/\sqrt{5})$

3. Any two lines in S^2 ______.

   A. intersect at a point

   B. intersect at exactly two points

   C. are parallel

   D. are perpendicular

4. Which of the following is a pole of a line that is perpendicular to the line ℓ in S^2 which has pole $\psi = (1/\sqrt{2}, 0, -1/\sqrt{2})$?

   A. $(-1/\sqrt{2}, 0, -1/\sqrt{2})$ C. $(1/\sqrt{2}, 1, 1/\sqrt{2})$

   B. $(0, 1/\sqrt{2}, -1/\sqrt{2})$ D. $(1, 0, -1/\sqrt{2})$

5. Which of the following are the points of intersection of the lines m and n in S^2 having poles ψ_1 and ψ_2, respectively?

   A. $\pm(\psi_1)/\langle\psi_1, \psi_2\rangle$

   B. $\pm(\psi_2)/\langle\psi_1, \psi_2\rangle$

   C. $\pm(\psi_1 \times \psi_2)/\langle\psi_1, \psi_2\rangle$

   D. $\pm(\psi_1 \times \psi_2)/|\psi_1 \times \psi_2|$

6. Which of the following describes the plane containing the noncollinear points P, Q, and R?

   A. $Q + [P - Q, P - R]$ C. $P + [Q - P, R \times P]$

   B. $Q + [P - Q]$ D. $P + [Q \times R]$

7. Which of the following motions in S^2 are equivalent?

   A. reflection and rotation

   B. rotation and translation

   C. reflection and translation

8. In S^2, how many lines can pass through P and $-P$?

   A. 0 C. 2

   B. 1 D. infinitely many

9. Which of the following is NOT an essential property of a plane Π in $\mathbb{E}^3$?

   A. $\Pi \neq \mathbb{E}^3$

   B. $P, Q \in \Pi$ implies that line $PQ \in \Pi$

   C. Every triple P, Q, R is on Π.

   D. $\Pi \notin \ell$ for all lines $\ell \subseteq \mathbb{E}^3$

10. Which of the following is orthogonal to each of the vectors v and w in $\mathbb{R}^3$?

    A. $v + w$ C. $v \times w$

    B. $v \cdot w$ D. $\langle v, w\rangle$

11. If $\{u, v, w\}$ is an orthonormal basis for $\mathbb{R}^3$. Then for any vector $x \in \mathbb{R}^3$, we have ______.

    A. $x = \langle x, w\rangle w + \langle x, v\rangle v + \langle x, u\rangle u$

    B. $x = \langle x, v\rangle w + \langle x, v\rangle u + \langle x, u\rangle v$

    C. $x = \langle v, w\rangle w + \langle u, v\rangle v + \langle w, u\rangle u$

    D. $x = \langle x, w\rangle x + \langle x, v\rangle x + \langle x, u\rangle x$

12. Which of the following statements is TRUE?

    A. The product of two reflections is also a reflection.

    B. Every isometry is injective.

    C. The product of any three reflections is a reflection.

    D. The identity belongs to the set of all reflections.

13. The angle of rotation of $\Omega_m\Omega_\ell$ where m is $x+y+6 = 0$ and ℓ is the line $2x-2y+1 = 0$ is equal to ______.

    A. $\pi/4$ C. $\pi/2$

    B. $\pi/3$ D. π

14. If the fixed point set of an isometry T is the whole plane $\mathbb{E}^2$, then T is ______.

    A. the identity C. a glide reflection

    B. a rotation D. a reflection

15. In the formula $\Omega_\ell X = X - 2\langle X - P, N\rangle N$, the vector N is ______.

    A. a unit normal vector to ℓ

    B. a unit direction vector of ℓ

    C. a point on ℓ

    D. a point orthogonal to X

16. Which of the following has the identity map I as one of its special types?

A. reflection
B. translation
C. glide reflection
D. half turn

17. All of the following represent the direction containing the vector $(-2,5)$ EXCEPT ______.

A. $[(-2,5)]$
B. $[(2,-5)]$
C. $[(5,-2)]$
D. $[(1,-5/2)]$

18. All of the following are points on the line $\ell = (-2,1)+[(3,-5)]$ EXCEPT ______.

A. $(-2,1)$
B. $(3,-5)$
C. $(1,-4)$
D. $(-8,11)$

19. Which of the following is TRUE if $P+[v]=Q+[w]$?

A. $P=Q$
B. $v=w$
C. $[v]=[w]$
D. All of these

20. Which of the following represents the y-axis?

A. $(0,\pi)+[(0,-\sqrt{2})]$
B. $(\pi,0)+[(0,-\sqrt{2})]$
C. $(0,\pi)+[(-\sqrt{2},0)]$
D. $(\pi,0)+[(-\sqrt{2},0)]$

For numbers (21) - (22), let $P=(1,2)$ and $Q=(5,-1)$.

21. What is $d(P,Q)$?

A. $\sqrt{7}$
B. $\sqrt{37}$
C. 5
D. 25

22. Which of the following represents the line through P and Q?

A. $(1,2)+[(5,-1)]$
B. $(5,-1)+[(6,1)]$
C. $(5,-1)+[(1,2)]$
D. $(1,2)+[(4,-3)]$

23. Which of the following is an orthonormal pair?

A. $\{(2,-1),(1,-2)\}$
B. $\left\{\left(\frac{2}{5},-\frac{\sqrt{21}}{5}\right),\left(-\frac{\sqrt{21}}{5},-\frac{2}{5}\right)\right\}$
C. $\left\{\left(-\frac{4}{\sqrt{5}},-\frac{\sqrt{3}}{4}\right),\left(\frac{\sqrt{3}}{4},-\frac{4}{\sqrt{5}}\right)\right\}$
D. $\left\{\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right),\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)\right\}$

24. Let ℓ be the line passing through P and parallel to the line passing through the points Q and R. If X is a point on ℓ, then for some $t \in \mathbb{R}$, X can be written as ______.

A. $X=P+t(Q-P)$
B. $X=P+t(R-Q)$
C. $X=P+t(Q-P)^{\perp}$
D. $X=P+t(R-Q)^{\perp}$

25. Which of the following is NOT a normal vector to the line $2x-3y+6$?

A. $(-2,-3)$
B. $(2/3,-1)$
C. $(-2,3)$
D. $(1,-3/2)$

26. Let ℓ be the line $2x+y=1$ and $X=(1,0)$. If m is the unique line through X perpendicular to ℓ, and F is the intersection point of ℓ and m, then the distance between X and F is ______.

A. $1/\sqrt{5}$
B. $2/\sqrt{5}$
C. $\sqrt{5}$
D. $2\sqrt{5}$

27. The mapping $\Omega_m\Omega_n$ is a trivial translation if ______.

A. $m=n$
B. $m\perp n$
C. $m \parallel n$
D. $m\cap n \neq \emptyset$

28. Which of the following statements is FALSE?

A. If $\wp$ is the pencil of parallels with common perpendicular $\ell = P+[v]$, then every line $m \in \wp$ has direction $[v^{\perp}]$.
B. The identity is an element of the group $TRANS(\ell)$.
C. A direction of the line with equation $ax+by=c$ is $[(a,b)]$.
D. Every half turn has a pencil of fixed lines.

For numbers (29) - (33), let $x,y,z \in \mathbb{R}^2$ and $c \in \mathbb{R}$

29. All of the following are scalars EXCEPT ______.

A. $x+y$
B. $\langle x,y\rangle$
C. $|x+y|$
D. $d(x,y)$

30. Which of the following statements is NOT true?

A. $\langle x,cy\rangle = \langle cx,y\rangle$
B. If $\langle x,y\rangle = 0, \forall x \in \mathbb{R}^2$, then $y=0$.
C. $\langle x,y\rangle + \langle x,z\rangle = \langle 2x, y+z\rangle$
D. If $y=0$, then $\langle x,y\rangle = 0, \forall x \in \mathbb{R}^2$.

31. Which of the following is equal to $\langle cx,cy\rangle$?

A. $c\langle x,y\rangle$
B. $\langle cx,y\rangle$
C. $\langle x,cy\rangle$
D. $c^2\langle x,y\rangle$

32. Which of the following vectors is NOT proportional to $(1, -3/2)$?

A. $(-2/3, 1)$
B. $(2/3, -1)$
C. $(2, -3)$
D. $(3, 2)$

33. Which of the following is TRUE for all $x \in \mathbb{R}^2$ and $c \in \mathbb{R}$?

A. $\langle x, x \rangle = |x|^2$
B. $|x| > 0$
C. $|cx| = c|x|$
D. All of these

34. Let α, β, γ, and δ be four distinct lines in the pencil of parallels $\wp$. Then $\Omega_\alpha \Omega_\beta \Omega_\gamma \Omega_\delta$ is a _______.

A. reflection
B. translation
C. rotation
D. glide reflection

35. The image of any point P with respect to rot 2π is _______.

A. $P^\perp$
B. $-P^\perp$
C. P
D. $-P$

36. If α, β, and γ are three distinct lines that are not concurrent and not all parallel, then $\Omega_\alpha \Omega_\beta \Omega_\gamma$ is a nontrivial _______.

A. reflection
B. translation
C. rotation
D. glide reflection

37. If T is an isometry whose set of fixed points is empty, then T is _______.

A. the identity
B. a rotation
C. a reflection
D. a glide reflection

38. Which of the following statements is TRUE?

A. If ℓ is a fixed line of an isometry T, then every point X on ℓ is also fixed.
B. A half turn does not have any fixed line.
C. A trivial reflection is the identity.
D. The identity is also a translation.

39. Which of the following can have only one fixed point?

A. the identity
B. a rotation
C. a reflection
D. a glide reflection

40. If u, v is an orthonormal pair of vectors in $\mathbb{R}^2$, then _______.

A. $|u| = |v| = 1$ and $\langle u, v \rangle = 0$
B. $|u| = |v| = 1$ and $\langle u, v \rangle = 1$
C. $|u| = |v| = 0$ and $\langle u, v \rangle = 1$
D. $|u| = |v| = 0$ and $\langle u, v \rangle = 0$

41. Which of the following is TRUE about the line $\ell = P + [v]$?

A. It contains all points of the form $t(P - v)$, where $t \in \mathbb{R}$.
B. It contains all points of the form $v + tP$, where $t \in \mathbb{R}$.
C. It has a normal vector equal to $-v^\perp$.
D. The set $[v^\perp]$ is also its direction.

42. Which of the following is an abelian group?

A. $REF(P)$ - the set of all reflections along lines that pass through P
B. $O(2)$ - the set of all rotations about the origin and all reflections along lines that pass through the origin
C. $TRANS(\ell)$ - the set of all translations along line ℓ
D. $\mathcal{I}(\mathbb{E}^2)$ - the set of all isometries in $\mathbb{E}^2$

43. Which of the following isometries has a line of fixed points?

A. glide reflection
B. translation
C. rotation
D. reflection

44. Which of the following has no fixed points?

A. identity
B. nontrivial translation
C. nontrivial rotation
D. reflection

45. Each of the following may have a pencil of fixed lines EXCEPT _______.

A. glide reflection
B. translation
C. rotation
D. reflection

46. The equality $\Omega_\ell \Omega_m = \Omega_m \Omega_\ell$ is true if _______.

A. $\ell \perp m$
B. $\ell \parallel m$
C. $\ell \neq m$
D. $\ell \cup m \neq \emptyset$

47. Let ℓ be the line $x + y = 0$ and m be the line $x - 3y + 3 = 0$. Then the product $\Omega_\ell \Omega_m$ is a _______.

A. nontrivial translation
B. nontrivial rotation
C. glide reflection
D. reflection

48. Which of the following is involutive (i.e., a map T such that $T^2 = I$)?

    A. reflection
    B. nontrivial translation
    C. glide reflection
    D. nontrivial rotation

49. The product of any two distinct half-turns is a ______.

    A. reflection
    B. nontrivial translation
    C. glide reflection
    D. nontrivial rotation

50. The image of the point $(2, -1)$ under the half-turn H_P, where $P = (-1, 3)$ is the center of rotation is ______.

    A. $(-1, 2)$
    B. $(-2, 1)$
    C. $(-4, 7)$
    D. $(-4, 3)$

51. The product $\Omega_\alpha \Omega_\beta \Omega_\gamma$ is a nontrivial glide reflection if ______.

    A. α, β, and γ are three distinct parallel lines.
    B. α, β, and γ are three distinct concurrent lines.
    C. α, β, and γ are three distinct lines which are nonconcurrent and not all parallel. lines.

For numbers (52) - (55), let $u = (-15, 20)$ and $v = (-2, -1)$.

52. What is $u - v$?

    A. $(-17, 19)$
    B. $(17, -19)$
    C. $(13, -21)$
    D. $(-13, 21)$

53. What is $|u|$?

    A. 5
    B. $5\sqrt{7}$
    C. 25
    D. 625

54. What is $\langle u, v\rangle$?

    A. -50
    B. -10
    C. 10
    D. 50

55. What is $\langle v, v\rangle$?

    A. $\sqrt{5}$
    B. $\sqrt{3}$
    C. 5
    D. 3

56. A half turn in S_2 has ____ fixed points.

    A. 0
    B. 1
    C. 2
    D. infinitely many

57. The antipodal map leaves ____ line(s) fixed.

    A. no
    B. one
    C. two
    D. all

58. Every rotation in S^2 can be written as a product of two ____

    A. antipodal maps
    B. half turns
    C. translations
    D. all of the above

59. Let P, Q, and X be distinct points of S^2. If P and Q are not antipodal, a point X lies on the minor segment of PQ if and only if ____

    A. $d(P, X) + d(X, Q) = d(P, Q)$
    B. $d(P, X) + d(X, Q) > d(P, Q)$
    C. $d(P, X) + d(X, Q) \geq d(P, Q)$
    D. $d(P, X) + d(X, Q) \leq d(P, Q)$

60. Which of the following matrices could represent a glide reflection in S^2?

    A. $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \end{bmatrix}$

    B. $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

    C. $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & -1 \end{bmatrix}$

    D. $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

END OF EXAM

NUMERICAL ANALYSIS **Sample Examination**

Directions: Answer the following as indicated and in the given order. Show your complete solution in each case.

Part I: Multiple Choice. *Write the letter of the correct answer in your answer sheet. Be sure to fill in the ovals completely. If the answer is not among the given choices, write E.* *(2 points each)*

1. The function $f(x) = \sin x - e^x$ has a root on the interval ____.

   A. $[-4, -3]$ B. $[-3, -2]$ C. $[0, 1]$ D. $[1, 2]$

2. If x_n is the nth estimate of the root s of the given function f using the Bisection Method, which of the following is always true?

   A. $\lim_{n\to+\infty} |x_n - s| = 0$
   B. $x_n = s$
   C. $\lim_{n\to 0} |x_n - s| = 0$
   D. $f(x_n) = s$

3. The auxiliary function $g(s)$ of a continuous function $f(x)$ is a solution to the function $f(x)$ if

   A. s is a root of f.
   B. s is a root of g
   C. $g(s) = f(s)$.
   D. $g(s) = f(x)$ for all x.

4. For an equation like $x^2 = 0$, a root exists at $x = 0$. The bisection method cannot be adopted to solve this equation in spite of the root existing at $x = 0$ because the function $f(x) = x^2$

   A. is a polynomial
   B. has repeated roots at $x = 0$
   C. is always non-negative
   D. has a slope of zero at $x = 0$

5. Which method may be used to estimate the root of a given function differentiable at each point on the given interval?

   A. Bisection Method
   B. Fixed Point Method
   C. Both (A) and (B)
   D. Neither (A) nor (B)

6. If $P(x)$ is an interpolating polynomial of the function $f(x)$ on $[a, b]$, then for each node x_i on the interval

   A. $P'(x_i) = f'(x_i)$
   B. $P'(c) \approx f'(c), a < c < b$
   C. $P'(x)$ has a root on $[a, b]$
   D. $P'(x)$ can be factored

7. Let $AX = B$ be a linear system of n equations in n unknowns. If A is diagonally dominant, which method may be used to find an estimate of the solution of the linear system?

   A. Gauss-Jordan Method
   B. Gauss-Jacobi Method
   C. Gauss-Seidel Method
   D. All of these

8. Given a difference table for a given set of interpolation points $(x_0, \ldots, x_n)$ with common difference h, which method is the most appropriate to find an estimate for $f(x_0 + k), k < h$?

   A. Method of Divided Differences
   B. Newton's Forward Difference Formula
   C. Newton's Backward Difference Formula
   D. All of these

For numbers 9 and 10, consider the LU decomposition of the matrix given below

$$\begin{bmatrix} 3 & 1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$$

9. The upper triangular matrix U is

   A. $\begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ D. $\begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$

10. The lower triangular matrix L is

    A. $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$
    B. $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$
    C. $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$
    D. $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

11. If the Romberg quadrature of order 4 is used to approximate a definite integral, then the largest number of subintervals used to generate the estimates $T_{i,0}$ is

    A. 4 B. 8 C. 16 D. 32

12. If a Gaussian quadrature is to be exact for all polynomials of degree less than or equal to 7, then the least number of nodes that must be used is

    A. 3 B. 4 C. 5 D. 6

13. For the improper integral $\int_0^{+\infty} \frac{xe^{-x}}{\sqrt{x+1}}\, dx$, which Gaussian quadrature is applicable?

    A. Gauss-Chebyshev
    B. Gauss-Hermite
    C. Gauss-Laguerre
    D. Gauss-Legendre

14. If $n = 12$ subintervals will be used, which composite quadrature can you use?

    A. Simpson's 1/3 rule
    B. Simpson's 3/8 rule
    C. both A and B
    D. neither A nor B

15. If an estimate for $y(1.6)$ is desired for an initial value problem $y' = f(t, y), y(0) = y_0$ using a step-size of $h = 0.2$, how many iterations of a one-step method are required?

    A. 3 B. 6 C. 8 D. 16

16. The modified Euler's formula tries to approximate the degree of accuracy of the estimate provided by the

    A. Taylor series method of order two
    B. Runge-Kutta method of order four
    C. 2-step Adams formulas
    D. Milne's method

17. For the initial value problem $(x^2 - 3xy)dx + 4y^2dy = 0,\ y(0) = 2$, $f(x,y)$ is equal to

A. $x^2 - 3xy$

B. $\dfrac{3xy - x^2}{4y^2}$

C. $4y^2$

D. $\dfrac{4y^2}{x^2 - 3xy}$

18. If the shifted inverse power method is applied to the matrix $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & -4 & 2 \\ 0 & 0 & -1 \end{bmatrix}$ with $\alpha = 1.8$, then estimates can be obtained for which eigenvalue?

A. -1 B. 0 C. 2 D. -4

19. For the preceding question, which numerical method is the most appropriate to use to obtain estimates for the eigenvalue $\lambda = 2$?

A. Power method

B. Inverse power method

C. Shifted inverse power method

D. all of the above

20. If the power method is used and $Y^{(3)} = (4\ -3\ -5)^T$, then c_4 is equal to

A. 5 B. 4 C. 3 D. -5

Part II: Problem Solving. *Use the indicated numerical method to solve the given approximation problem. Carry your computations to eight decimal places.*

1. Use ten iterations of the Newton (or Newton-Raphson) Method to find an estimate for a root of the function $f(x) = x\cos 4x$. Use $x_0 = 0.75$. (8 *Points*)

2. With $X^{(0)} = (0,0,0,0)$, use ten iterations of the Gauss-Seidel Method to solve the system. (8 *Points*)

$$\begin{aligned} x + 3y \quad + w &= 11 \\ 8x + 2y + z + 3w &= 27 \\ 2x - 2y - z - 6w &= -29 \\ -3x + 2y + 10z + 4w &= 43 \end{aligned}$$

3. Given the points $(-1,5), (1,-5)$ and $(2,-1)$,

   (a) Find the quadratic polynomial which interpolates these points using the Lagrange Form of the Interpolating Polynomial. (8 *Points*)

   (b) From the interpolating polynomial you obtained in number 3, find an estimate of $f'(0.5)$. (5 *Points*)

4. Find an estimate for $\displaystyle\int_0^3 \frac{dx}{9+x^2}$ using Simpson's 1/3 rule with $h = 0.25$. Compute the absolute error of the estimate. (*8 points*)

5. Use the 3-step Adams formulas and a step-size of $h = 0.1$ to obtain an estimate for $y(0.6)$ for the initial value problem $y' = \dfrac{t^2}{y},\ y(0) = 1$. Use the Runge-Kutta method of order 4 to generate the required number of starting values. Compute the absolute error. (*12 points*)

6. Consider the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$. Use as many iterations of the power method as necessary to obtain estimates for the dominant eigenvalue and a dominant eigenvector using $X^{(0)} = (1,1,1)^T$ as the initial estimate, such that $\|X^{(n)} - X^{(n-1)}\| \le 10^{-7}$. (*8 points*)

7. Use six iterations of the shifted inverse power method to obtain estimates for the eigenpairs of the matrix $A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$. (*12 points*)

FORMULAS

$P(x) = \sum_{j=0}^{n} f(x_j) l_j(x)$

where

$$l_j(x) = \frac{(x-x_0)(x-x_1)\ldots(x-x_{j-1})(x-x_{j+1})\ldots(x-x_n)}{(x_j-x_0)(x_j-x_1)\ldots(x_j-x_{j-1})(x_j-x_{j+1})\ldots(x_j-x_n)}$$

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

$$x_i^{(k)} = \frac{-\sum_{j=1}^{i-1} a_{ij}x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k-1)} + b_i}{a_{ii}}$$

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i\right]$$

$$y_{j+1} = y_j + \frac{h}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right] \text{ where}$$

$$k_1 = f(x_j, y_j),\ k_2 = f\left(x_j + \frac{h}{2}, y_j + \frac{h}{2}k_1\right),$$

$$k_3 = f\left(x_j + \frac{h}{2}, y_j + \frac{h}{2}k_2\right),\ k_4 = f(x_j + h, y_j + hk_3)$$

$$y_{j+1}^{(p)} = y_j + \frac{h}{12}(23f_j - 16f_{j-1} + 5f_{j-2})$$

$$y_{j+1}^{(c)} = y_j + \frac{h}{12}(5f_{j+1} + 8f_j - f_{j-1})$$

Instruction: Answer the following as indicated. In each case, show your complete solution. This exam has two (2) pages.

1. Formulate an LP or an IP for the following problems. Please define all variables and constraints carefully. [5 points each]

   (a) The Bayumbong Chamber of Commerce periodically sponsors public service seminars and programs. Currently, promotional plans are under way for this year's program. Advertising alternatives include television, radio, and newspaper. Audience estimates, costs, and maximum media usage limits are as shown:

Constraint	Television	Radio	Newspaper
Audience per advertisement	100,000	18,000	40,000
Cost per advertisement	P2,000	P300	P600
Maximum media usage	10	20	10

   To ensure a balanced use of advertising media, radio advertisements must not exceed 50% of the total number of advertisements authorized. In addition, television should account for at least 10% of the total number of advertisements authorized. How many adverstisements in each medium should be allocated so as to maximize the number of audience reached if the promotional budget is limited to P18,200?

   (b) I bought a triple sim cellular phone and I have been approached by three telecom companies to subscribe to their short messaging service (text messaging service). World telecoms will charge a flat rate of P200.00 per month plus P 0.50 per sms . Intelligent telecoms will charge P300.00 per month but will reduce the per message to P0.30. As for hug mobile , the flat monthly charge is P250.00 and the cost per message is P0.35. I usually send an average of 1,500 messages a month. Assuming that I don't pay the flat rate unless I send messages and I can apportion my messages among all three companies as I please, how should I use the three companies to minimize my monthly bill?

2. Solve the following LP using primal simplex method. [5 points]

$$\begin{aligned} Maximize\ z &= 4x_1 + 3x_2 + 6x_3 \\ subject\ to & \\ 3x_1 + x_2 + 3x_3 &\leq 30 \\ 2x_1 + 2x_2 + 3x_3 &\leq 40 \\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

3. Given the following linear programming model

$$\begin{aligned} Maximize\ z &= 3x_1 + 4x_2 & \\ subject\ to & & \\ 2x_1 + 3x_2 &\leq 1200 & Resource\ 1 \\ 2x_1 + x_2 &\leq 1000 & Resource\ 2 \\ 4x_2, &\leq 800 & Resource\ 3 \\ x_1,\ x_2 &\geq 0 & \end{aligned}$$

(a) Construct the dual of the model. [3 points]

(b) Complete the optimal simplex tableau of the LP. [4 points]

Basic	x_1	x_2	x_3	x_4	x_5	Solution
z	0	0			0	
x_1	1	0	$-\frac{1}{4}$	$\frac{3}{4}$	0	
x_5	0	0	-2	2	1	
x_2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	

(c) Determine the status and unit worth of each resource. [3 points]

(d) If the available number of resource 1 is increased to 1600 units and resource3 is decreased to 500 units and resource 2 remains the same. Find the new optimum solution. Is it still feasible? If not, how do we "‘remedy"’ the situation? [3 points]

(e) Will the optimal solution change if the unit profit of model 2 is decreased from 4 to 3 and the unit profit of model 1 remains the same. Why? What about the objective value? [3 points]

(f) Will the optimal solution change if the constraint $x_1 + 5x_2 \leq 100$ is added to the model. Why? [3 points]

4. Answer the following completely. [2 points each]

(a) Give the relationship between the extreme points of a feasible region and the basic feasible solutions of the simplex algorithm.

(b) What is the advantage of the Two-Phase Method over the Big M Method?

(c) Why is it that infeasibility can never occur if all the constraints of the model are of the type $\leq$?

(d) In the optimal tableau of the Big M method, what conclusion can you draw if an artificial variable is left at the basis at zero level?

5. Write True if the statement is always true otherwise write False and justify your answer. [2 points each]

(a) For any pair of feasible primal and dual solutions, the objective value in the minimization problem is less than or equal to the objective value in the maximization problem.

(b) An unrestricted variable in the primal model always produces an equality constraint in the dual model.

(c) If the dual variable $y_i = 0$ then it is worth increasing resource i to have a better objective value.

(d) If one if the coefficients of the objective function of an LP is changed then this will always affect the optimality of the current solution.

End of Exam
Total : 50 points

1. Given the sample space $\Omega = \{ 1, 2, 3, 4 \}$
   1.1 Give the smallest algebra containing the event $A = \{ 2 \}$.
   1.2 Give the largest possible algebra corresponding to this sample space.
   1.3 Is the set of events $\mathcal{F} = \{ \phi, \Omega, \{ 3 \}, \{ 1, 2 \}, \{ 3, 4 \}, \{ 1, 2, 4 \} \}$ an algebra? Why or why not?

   If not, give the missing elements to make it an algebra.

2. Discuss briefly the relationship between the Poisson and Exponential distributions.

3. Prove that if an event A is independent of itself, then P(A) = 0 or 1.

4. Given P[A] = 0.3 and P[B] = 0.2, find
   4.1 $P[A \cap B]$ if $P[A \cup B] = 0.05$
   4.2 $P[A \cup B]$ if A and B are mutually exclusive events
   4.3 $P[A \cup B]$ if A and B are independent events\
   4.4 P[A/B] if A and B are mutually exclusive events
   4.5 P[A/B] if A and B are independent events.

5. Given 10 examination papers no two of which have the same score. If the papers are randomly arranged, what is the probability that the best (highest score) and worst (lowest score) papers are
   5.1 together?
   5.2 not together?

6. Let X = the number of letters in a word chosen at random from the following quotation by Sir Albert Einstein:

"God does not play dice with the universe."

   6.1 Give the pmf of X in tabular form.
   6.2 Give the mean and variance of X.
   6.3 Give the cdf of X.

7. A random variable X is said to have a Poisson distribution if it has the pmf $f_X(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \dots$,

   where λ is a positive constant.

   7.1 Show that the mgf of X is given by $m_X(t) = e^{-\lambda + \lambda e^t}$. (*Hint:* $e^r = \sum_{x=0}^{\infty} \frac{r^x}{x!}$)
   7.2 Use the mgf of X from (7.1) to show that $E[X] = \lambda$ and $Var[X] = \lambda$.
   7.3 Find P[X = 4] if it is given that P[X = 1] = P[X = 2].

8. Given the following cdf of the continuous random variable X, $F_X(x) = \left(1 - e^{-x}\right) I_{[0,\infty)}(x)$.

   8.1 Give the pdf of X, $f_X(x)$.
   8.2 Find the median of X.
   8.3 Find P[$e^X \geq 2$].

More on Page 2

9 Records show that the number of deaths among patients residing in a large nursing home is a Poisson random variable with a rate of 0.2 per day.

9.1 Find the probability of at least 1 death in one day.

9.2 What is the probability that the next two deaths will occur within 4 days of one another?

9.3 If Y = number of days during the next 10 days with at least 1 death per day, refer to your answer in (9.1) and find the probability that there will be at least 1 death per day in exactly 3 of the next 10 days.

10. The employees of a firm that manufactures insulation are being tested for indications of asbestos in their lungs. The firm is requested to send 3 employees who have positive indications of asbestos to a medical center for further testing. If 40% of the firm's employees have positive indications of asbestos in their lungs,

10.1 find the probability that 10 employees must be tested in order to find 3 positives.

10.2 give the probability that out of 15 employees tested, exactly 3 positives will be found.

10.3 rework (10.2) using normal approximation.

10.4 If instead of the given 40%, only 4% of the firm's employees have positive indications of asbestos in their lungs, rework (10.2) using Poisson approximation.

11. Let the random variable X denote a person's intelligence quotient (IQ), which can be assumed to be normally distributed with a mean of μ = 100 and a standard deviation of σ = 16.

11.1 What proportion of the general population have IQs below 90?

11.2 MENSA (from the Latin word for "*mind*") is an international society devoted to intellectual pursuits. Any person who has an IQ in the upper 2% of the general population is eligible to join. What is the lowest IQ that will qualify a person for membership?

12. Let (X,Y) have the joint pdf given by $f_{X,Y}(x,y) = \begin{cases} k, & 0 \le x \le 2,\ 0 \le y \le 1,\ 2y \le x \\ 0, & otherwise \end{cases}$.

Find k.

13. Consider (X,Y) with joint probability density function $f_{X,Y}(x,y) = \begin{cases} \frac{3}{4}x, & 0 < x < 2;\ 0 < y < 2-x \\ 0, & otherwise \end{cases}$.

13.1 Give the marginal pdf of X, $f_X(x)$.

13.2 Give the cumulative distribution function of X, $F_X(x)$.

13.3 Find P[Y < 0.5 | X <1.5].

14. Let X and Y be discrete random variables with joint probability function (joint pmf):

$f_{X,Y}(x,y)$		X		
		1	2	4
Y	2	2/24	1/24	0
	4	4/24	2/24	1/24
	8	8/24	4/24	2/24

14.1 Give $P\left[X + \frac{Y}{2} \le 5\right]$.

14.2 Give the marginal pmf of X, $f_X(x)$ in tabular form.

14.3 Give the marginal pmf of Y, $f_Y(y)$ in tabular form.

14.4 Are X and Y independent? Justify.

14.5 $E[Y \mid X = 2]$.

Good Luck!

MULTIPLE CHOICE. Write the letter corresponding to the best answer in your test booklet. If the answer in not among the choices, write "E".

1. The collection and presentation of data and some computations of measures of characteristics like measures of central tendency, dispersion, skewness and kurtosis belong to the branch of statistics called _____ statistics.
   A. descriptive
   B. theoretical
   C. inductive
   D. inferential

For numbers 2 and 3, consider the following.
"I'm so stressed" is a common cry among college students. What is stressing you out?

2. Which of the following is(are) TRUE?
   A. The population of interest is the set of stressed college students.
   B. A sample has to be taken because it would take too much time and cost too much money to study the entire population.
   C. Both A and B
   D. Neither A nor B

3. What is the variable of interest?
   A. whether or not the student is stressed out
   B. whether or not he is a college student
   C. whether the student cries or not
   D. sources of stress

For numbers 4 to 6, consider the following:
The administrators of a university are trying to determine whether they have an adequate amount of parking. They take a random sample of 25 entering freshmen and ask them what mode of transportation thy will use to travel to classes. Here are the responses.

Car	Car	Car	Train	Bus
Bus	Bus	Bus	Car	Train
Train	Train	Car	Walk	Walk
Bus	Car	Train	Train	Car
Car	Bus	Bus	Bus	Walk

4. Which statement is TRUE about the variable of interest?
   A. It is qualitative.
   B. It is in the ordinal level of measurement.
   C. Both A and B
   D. Neither A nor B

5. Which statement is TRUE?
   A. The only measure of central tendency appropriate to summarize the data is the mean.
   B. The least popular mode of transportation is the train.
   C. The mode of transportation that is most popular is the bus.
   D. The data shows that there are two modes of transportation: private and public.

6. From the given data, which graph is most appropriate to represent the proportions or percentages of students using these modes of transportations?
   A. line graph B. boxplot C. stemplot D. pie graph

7. The starting incomes for a sample of newly graduates from a particular university was recorded and summarized in the following frequency distribution table:

Starting Salary (in PhP1000)	*Frequency*
10-14	3
15-19	5
20-24	10
25-29	6
30-34	2

   If a starting salary of P25,000 is considered as high, what percentage of these graduates have high salaries?
   A. 8 B. 23 C. 31 D. 69

For numbers 8 and 9, consider the following.

Grading homework is a real problem and takes an enormous amount of time. A teacher decides to conduct a study to determine what effect grading homework had on her students' exam scores. She randomly assigned each class to one of two conditions: no homework given and with homework given, collected, and graded. Scores on the first exam were summarized below for each class.

Summary Measure	No Homework	Homework Collected and Graded
Mean	76.6	83.5
Median	77.5	83.0
Range	45	32
Standard Deviation	10.1	8.0

8. Which of the following is true?
   A. On the average, the performance of students who were and were not given homework are the same.
   B. On the average, students who were not given homework performed better in the first exam than students who were given any homework.
   C. On the average, students who wore given homework performed better in the first exam than students who were not given any homework.
   D. No conclusions can be made regarding the performance of the students in the two classes.

9. Which of the following is(are) TRUE about the variability of the scores in the first exam.
   A. The distribution of the scores of students who were given no homework is less variable.
   B. The distribution of the scores of students who were given homework is less variable.
   C. The dispersion of the distributions of the scores of the students who were and were not given homework is the same.
   D. No conclusions can be made regarding the dispersion of the distributions of the scores in the two classes.

10. You were told that you got a percentile rank of 90% in a college entrance test. Which is an implication of this statistic?
    A. You belong to the top 10% of those who took DLSU college entrance test.
    B. You got a score of 90% in the DLSU college entrance test.
    C. Your score is above the median but below the third quartile.
    D. Your score is an outlier.

For numbers 11 and 12, consider the following:

The numbers of hours spent in the computer lab by 20 students working on a thesis are given below:

Number of Hours:

30	0 2 5 5 6 6 7 8	Leaf Unit: 1
40	0 2 2 5 7 9	
50	0 1 3 5	
60	1 3	

11. The range for this data set is _____.
    A. 300 B. 303 C. 600 D. 603

12. The distribution of the numbers of hours these 20 students spent in the computer lab for their theses is _____.
    A. skewed to the left
    B. skewed to the right
    C. perfect symmetric
    D. almost symmetric

13. A drug for the relief of asthma can be purchased from 5 different manufacturers in liquid, tablet or capsule form, all of which come in regular and extra strength. In how many different ways can a doctor prescribe the drug for a patient suffering from asthma?
    A. 6
    B. 10
    C. 15
    D. 30

14. The 'Eating Club' is hosting a make-your-own sundae at which the following are provided:

Ice Cream Flavor	Toppings
chocolate	caramel
cookies-n-cream	Hot fudge
strawberry	marshmallow
vanilla	M&Ms
	nuts
	strawberries

How many sundaes are possible using 1 flavor of ice cream and 3 different toppings?

A. (4)(6)
C. $({}_4P_1)({}_6P_3)$
B. $({}_4C_1)({}_6C_3)$
D. $\frac{10!}{4!6!}$

15. In a survey of 120 students living in the dorms, 60 said they have desktop computers, 40 said they have computer laptops, and 20 said they have neither. What is the probability that the student has a computer laptop?
A. 1/6 B. 1/5 C. 1/4 D. 1/3

16. In a game called 'cara y cruz', 3 coins are tossed together. A player wins if the 3 coins land all heads or all tails. What is the probability of winning this game?
A. 1/2 B. 2/3 C. 1/4 D. 6/8

17. The Law of Large Numbers states that _____.
A. if a random sample of size n are drawn from a population, then as n approaches larger, the sampling distribution of the sample mean approaches the normal population, regardless of the form of the population distribution
B. the relative frequency probability of an event approaches the theoretical probability as the number of experiments increases
C. Both A and B
D. Neither A nor B

18. Consider a game of tossing two dice. You are considered lucky if you get same number on the two dice ("doubles") or a sum of 11. What is the probability that you will be declared lucky in this game?
A. 1/2 B. 1/6 C. 2/11 D. 2/9

For numbers 12 to 14, consider the following:

In a study of college students and binge drinking, researchers were interested in the number of times that a student had a hangover during the semester. The collected data were as follows:

	Hangover since beginning of semester			
Gender	Not at all	Once	Twice or more	Total
Male	61	23	40	124
Female	66	25	36	127
Total	127	48	76	251

A student is selected at random.

19. What is the probability that the student had a hangover since beginning of semester?
A. 48 / 251
C. 124 / 251
B. 76 / 251
D. 127 / 251

20. What is the probability that a student is a male and did not have a hangover during the semester?
A. 61 / 251
C. 127 / 251
B. 124 / 251
D. (124 / 251)*(127 / 251)

21. What is the probability that a student had two or more hangovers in a semester given that the student is a male?
A. 40 / 76
C. 40 / 251
B. 40 / 124
D. 40

22. A committee of size 5 is to be selected at random from 3 chemists and 5 physicists. If X is the number of physicists on the committee then X is a _________ random variable.
A. normal
C. hypergeometric
B. binomial
D. poisson

23. Tests for impurities commonly found in drinking water from private wells showed that 30% of all wells in a particular community have impurities. If a random sample of n large wells in the community are tested and X represents the number of wells found to have impurities, then X is a _________ random variable.
A. normal
C. hypergeometric
B. binomial
D. poisson

24. On the average, 8 persons per hour use an express teller machine situated inside a commercial complex. What is the probability that, during a randomly selected hour on a Friday afternoon, at most 1 person will use the teller machine?
A. 0.0027
C. 0.9970

B. 0.0030 D. 0.9997

25. According to the WTA Tour and ATP Tour, four of the top 10 women tennis players use Wilson rackets. Suppose two of these players have reached the finals of a tournament. Which of the following is(are)TRUE?
    A. The probability that exactly one finalist uses a Wilson racket is 0.1333.
    B. The probability that neither uses a Wilson racket is 0.3333.
    C. Both A and B
    D. Neither A nor B

26. Which statement is NOT true?
    A. The mean, median, and mode of a normal distribution are all equal.
    B. The total area under the normal curve is less than 1.
    C. The normal distribution is symmetrical about its mean.
    D. The standard normal distribution is specified by a mean 0 and a standard deviation 1.

27. The life of a brand of battery is normally distributed with a mean of 62 hours and a standard deviation of 6 hours. The probability that a single randomly selected battery will last for 55 to 65 hours is ______.
    A. 0.1217 B. 0.4302 C. 0.6915 D. 0.5698

28. The scores on a college entrance test are normally distributed with a mean of 400 and a standard deviation of 100. If this college will only consider applicants with scores in the upper 10 percent, what is the minimum score required for consideration at this university?
    A. 90 B. 272 C. 529 D. 600

29. If repeated samples of size 40 are taken from an infinite population, the distribution of sample means ______.

    A. always will be normal because we do not know the distribution of the population
    B. always will be normal because the sample mean is always normal
    C. will be approximately normal because the population is infinite
    D. will be approximately normal because of the central limit theorem

30. Which of the following is(are) TRUE about estimation?
    A. It is concerned with giving value(s) to unknown population parameter(s).
    B. Information provided by sample data is used in estimating the value(s) of unknown population parameters.
    C. Both A and B
    D. Neither A nor B

31. Which of the following is(are) TRUE?
    A. Increasing the level of confidence will result to a longer confidence interval.
    B. Increasing the sample size will result to a longer confidence interval.
    C. Both A and B
    D. Neither A nor B

For numbers 32 to 35, consider the following.

Is this basketball player really worth that much money? The number of points per game is a key in evaluating the contribution of an NBA player. The number of points scored per game for a sample of 38 games played by Shaquille O'Neal was recorded and summarized. Results showed that the mean number of points scored per game by O'Neal is 26.89 points with a standard deviation of 7 points. A 95% confidence interval estimate for the true average number of points scored per game by O'Neal is from 24.67 to 29.12 points.

32. The of the following is(are) TRUE?
    A. An estimate for the true average number of points scored per game by O'Neal is 26.89 points.
    B. An estimator for the true average number of points scored per game by O'Neal is the sample mean $\bar{x} = \frac{\sum_{i=1}^{n} X_i}{n}$.
    C. Both A and B
    D. Neither A nor B

33. Which of the following is(are) TRUE?
    A. The probability is 0.95 that the true average number of points scored per game by O'Neal is between 24.6675 to 29.1220 points.

B. To increase the probability that we have a "good" interval, widen the confidence interval.
C. Both A and B
D. Neither A nor B

34. What can we assert with 95% confidence about the possible size of our error *e* if we estimate the true average number of points scored per game by O'Neal to be 26.89 points?
A. 1.1364
B. 1.96
C. 2.2273
D. 4.4545

35. Based on the constructed confidence interval, which of the following is(are) TRUE if a "typical" NBA player scores has an average of 25 points per game?
A. O'Neal is a typical NBA player.
B. O'Neal is really worth that much money.
C. Both A and B
D. Neither A nor B

36. A survey is planned to determine the mean annual family medical expenses of employees of a large company. The management of the company wishes to be 99% confident that the sample mean is correct to within ±$50 of the true mean annual family medical expenses. A pilot study indicates that the standard deviation can be estimated at $400. How large a sample size is necessary?
A. 16
B. 21
C. 246
D. 425

37. Which of the following is(are) TRUE about tests of hypotheses?
A. It a statistical tool used to determine whether a statement about population parameter(s) is valid or not.
B. A researcher can be certain if a statement about the population parameter(s) is correct upon performing a test of hypothesis.
C. Both A and B
D. Neither A nor B

38. On your break at work, you usually get a soda from the vending machine. Each cup should contain 12 ounces of soda. You have been getting soda at this machine for a while and you think it is cheating you by either undefilling your cup. You want to know whether the machine is underfilling the cups. You formulated the following null and alternative hypotheses:

$$H_0: \mu = 12$$
$$H_1: \mu < 12$$

Which of the following is(are) TRUE about tests of hypotheses?
A. A type I error is committed when you complain to the vending machine company when in fact the machine did not need to be adjusted.
B. A type II error is committed when you do not complain to the vending machine company when in fact the machine needs to be adjusted.
C. Both A and B
D. Neither A nor B

For numbers 39 to 42, consider the following.

Studies have shown that the time required by workers to complete a certain manual operation follows the normal distribution with mean 26.6 minutes and a standard deviation of 3 minutes. A group of 25 workers was randomly chosen to receive a special training for 2 weeks. After the training, it was found that their average was 24 minutes. A test of hypothesis will be conducted to determine if the special training speeds up the operation?

39. Which is an appropriate alternative hypothesis?
A. $\mu > 24$
B. $\mu > 26.6$
C. $\mu < 24$
D. $\mu < 26.6$

40. Which is the most appropriate test to be used?
A. Z test for the mean, sigma known, using the test statistic $Z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$
B. t test for the mean, sigma unknown, using the test statistic $t = \dfrac{\bar{x} - \mu}{\dfrac{s}{\sqrt{n}}}$

C. Z test for the proportion using the test statistic $Z = \dfrac{\hat{p} - P}{\sqrt{\dfrac{PQ}{n}}}$

D. Chi-square test for the variance using the test statistic $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

41. What is the computed test statistic?
    A. –7.5056 C. 4.3333
    B. –4.3333 D. 7.5056

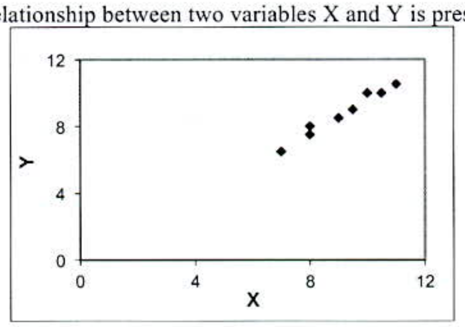
42. At α=5%, which of the following is(are) valid conclusion(s)?
    A. Do not reject H_0. The special training speeds up the operation.
    B. Do not reject H_0. The special training does not speed up the operation.
    C. Reject H_0. The special training speeds up the operation.
    D. Reject H_0. The special training does not speed up the operation.

43. Which graph is constructed to explore the relationship between two quantitative variables?

    A. pie chart C. boxplot
    B. histogram D. scatterplot

44. Which of the following is(are) TRUE about correlation coefficients?
    A. If the correlation coefficient is positive, then the slope of the estimated regression line is also positive.
    B. If two variables are perfectly linearly related, the correlation coefficient must be 1.
    C. Both A and B
    D. Neither A nor B

45. A graph to explore the relationship between two variables X and Y is presented below:



Which of the following is(are) TRUE?
    A. There is an inverse linear relationship between X and Y.
    B. There is no linear relationship between X and Y.
    C. There is a direct linear relationship between X and Y.
    D. The relationship between X and Y cannot be determined.

For numbers 46 and 47, consider the following.

A study (in the 1990's) on the relationship between fare and distance traveled showed that PhP0.75 is being added per kilometer traveled to an initial fare of PhP8.00.

46. Which of the following is(are) TRUE?
    A. Fare is the dependent variable. C. Both A and B
    B. Distance traveled is the independent variable. D. Neither A nor B

47. Which of the following is(are) TRUE about the model for the relationship between fare and distance traveled?
    A. The y-intercept is PhP8.00. C. Both A and B
    B. The slope is PhP0.75. D. Neither A nor B

For numbers 48 to 50, consider the following.

It was suspected that the monthly cost of maintenance (COST) of car in pesos seems to increase with the age (AGE) of the car in years. Data were collected from a random sample of 10 cars. Regression analysis was performed on the collected data, and the results of MS EXCEL are given below.

Regression Statistics	
Multiple R	0.9768839
R Square	0.95430216
Standard Error	73.67528
Observations	10

	Coefficients
Intercept	-1796.21849
AGE	2760.5042

48. Which of the following is(are) TRUE?
    A. There is a very strong direct linear relationship existing between the monthly cost of maintenance and the age of the car.
    B. 97.69% of the variation in the monthly cost of maintenance is explained by the age of the car.
    C. Both A and B
    D. Neither A nor B

49. Which of the following is(are) TRUE?
    A. The estimated monthly cost of maintenance of a 3 year old car is PhP6485.30.
    B. There is an increase of PhP2760.50 in the monthly cost of maintenance for every 1 year increase in the age of the car.
    C. Both A and B
    D. Neither A nor B

50. A test of hypothesis will be conducted to determine if monthly cost of maintenance increases with the age of the car. Which of the following is the appropriate alternative hypothesis?
    A. $\beta_1 = 0$ C. $\beta_1 \neq 0$
    B. $\beta_1 < 0$ D. $\beta_1 > 0$

PROBLEM SOLVING. Solve the following problems completely and as neatly as possible.

A. A manufacturer of jeans has plants in California (CA), Arizona (AZ), and Texas (TX). A group of 25 pairs of jeans is randomly selected from a department store, and the state in which each is produced is recorded: (10 points)

CA	AZ	AZ	TX	CA
CA	CA	TX	TX	TX
AZ	AZ	CA	AZ	TX
CA	AZ	TX	TX	TX
CA	AZ	AZ	CA	CA

1. Identify the variable of interest. Determine the type (qualitative) and level of measurement (nominal, ordinal, interval, ratio) of the identified variable.
2. Construct a one-way table for the given data and the most appropriate graph for it.

B. Consider the following stem and leaf display containing the viewers' rating of the quality of programming available on television. Respondents were asked to rate the overall quality from 0 (no quality at all) to 100 (extremely good quality). (15 points)

3	2 4
4	0 3 4 7 8 9 9 9
5	0 1 1 2 3 4 5
6	1 2 5 6 6
7	0 1
8	
9	1

Unit : 1

1. Construct a frequency distribution for the viewers' rating of the quality of programming available on television. Show complete solutions for R, k and c.
2. Give the five number summary and use it to construct a box plot for overall quality ratings.

C. The running times (in minutes) of a random sample of 5 films produced by motion-picture company A are 103, 94, 110, 87, and 98. (15 points)
   1. Find

a. mean
b. median
c. standard deviation
d. measure of skewness

2. A random sample of 5 films produced by another motion-picture company B has running times (in minutes) summarized below:

Summary Measure	*B*
Mean	105.2
Median	105
Standard Deviation	3.701351105
Skewness	0.162103

a. On the average, which motion-picture company produced longer films? Justify.
b. Which motion-picture company has a less variable/dispersed running times? Justify.
c. Which distribution of running times is symmetric? Justify.
d. Films produced each by the 2 companies are being shown in theaters. Based on your answers above, which film are you going to watch? Justify.

D. In an experiment involving a toxic substance, the probability that a white mouse will be alive for 10 hours is $\frac{7}{10}$, and the probability that a black mouse will be alive for 10 hours is $\frac{9}{10}$. Find the probability that, at the end of 10 hours: (5 points)
1. only the black mouse will be alive.
2. at least one mouse will be alive.

E. Brett and Margo have each thought about murdering their rich Uncle Basil in hopes of claiming their inheritance a bit early. Hoping to take advantage of Basil's fondness for immoderate desserts, Brett has put a rat poison in the cherries flambé; Margo, unaware of Brett's activities, has laced the chocolate mousse with cyanide. Given the amounts likely to be eaten, the probability of the rat poison being fatal is 0.60; the cyanide, 0.90. Based on other dinners where Basil was presented with the same dessert options, we can assume that he has a 50% chance of asking for the cherries flambé, a 40% chance of ordering the chocolate mousse, and a 10% chance of skipping dessert altogether. No sooner are the dishes cleared away when Basil drops dead. In the absence of any other evidence, who should be considered the prime suspect? Justify. (5 points)

F. You are trying to set up a portfolio that consists of a corporate bond fund and a common stock fund. The following information about the annual return (per $1,000) of each of these investments under different economic conditions is available, along with the probability that each of these economic conditions will occur:

Probability	State of the Economy	Corporate Bonds	Common Stocks
0.10	Recession	-$30	-$150
0.15	Stagnation	50	-20
0.35	Slow Growth	90	12
0.30	Moderate Growth	100	160
0.10	High Growth	110	250

Do you think you will invest in the corporate bond fund or the common stock fund? Justify your choice by computing the expected return for the corporate bond fund and the common stock fund. (10 points)

G. The Department of Commerce in a particular state has determined that the number of small businesses that declare bankruptcy per month has a mean of 6. It is of interest to determine the probability that more than two bankruptcies will occur next month. (10 points)
1. Define a discrete random variable X that is useful in determining the probability of interest.
2. Give the possible values of X.
3. Completely specify the probability distribution of X.
4. Find the probability that more than two bankruptcies occur next month.

H. Assume that the test scores from a college admissions test are normally distributed with a mean of 450 and standard deviation of 100. (15 points)
1. What percentage of the people taking the test score between 400 and 500?
2. If a particular university will not admit anyone scoring below 480, what percentage of the persons taking the test would be acceptable to the university?
3. If a particular university only admits the top 10% of those who took the college admissions test, what is the lowest possible score that a student should get in order to be considered for admission to this university?

I. A pollution control inspector suspected that a river community was releasing amounts of semitreated sewage into a river. To check his theory, he drew 10 randomly selected specimens of river water. The dissolved oxygen readings (in parts per million) are as follows:

5.0 5.2 5.0 4.9 5.1 4.8 5.0 5.1 5.3 4.9.

Perform the appropriate test to determine if the mean oxygen content exceeds 5 ppm. Use a 5% level of significance. (5 pts.)

J. The following data represent systolic blood pressure (SBP) readings on 15 females preselected by age (covering ages 40 to 85).

Age	**42**	**46**	**42**	**71**	**80**	**74**	**70**	**80**	**85**	**72**	**64**	**81**	**41**	**61**	**75**
SBP	130	115	148	100	156	162	151	156	162	158	155	160	125	150	165

1. Obtain the estimated linear regression equation. Use the obtained linear regression equation to predict the systolic blood pressure for a female of age 50. (5 pts.)
2. Compute the correlation coefficient and describe the relationship between systolic blood pressure and age. Also, compute for the sample determination coefficient R^2 and interpret. (5 pts.)

END OF EXAM

Directions: Answer the following as indicated and in the given order. In each case, show your complete solution.

1. **[10 pts]** A 1,000 bond with annual coupons is redeemable at par at the end of 10 years. At a purchase price of 877.11, the yield rate is i. The coupon rate is (i – 0.02) . Calculate i.

2. **[10 pts]** Suppose you buy a \$1000 par value bond maturing at par in 10 years and paying annual coupons at 4%, for a price to yield 8% effective. You deposit the coupon payments in a bank account paying 3% effective. What will be the annual effective yield rate on the investment?

3. **[10 pts]** A 1000 par value 10-year bond with annual coupons and redeemable at maturity at 1100 is purchased for P to yield an annual effective rate of 10% . The first coupon is 50. Each subsequent coupon is 5% greater than the preceding coupon. Determine P.

4. **[10 pts]** A machine is purchased for 120,000 and is expected to last 10 years. The book value of the machine at the end of the 3rd year is 87,480 by the declining balance method. Determine the absolute value of the difference between the book values at the end of the 5th year using the sum-of-the-digits method and the declining balance method.

5. **[10 pts]** A bicycle that costs 20,000 will have a salvage value of S at the end of 10 years. The periodic maintenance cost is 400, and the periodic charge is 600. The sinking fund rate and the yield rate are equal to 8% per annum. Calculate S.

6. **[10 pts]** The manager of ABC Company's Purchasing Department is comparing two machines with the following characteristics:

Machine	Cost	Annual Maintenance Expense	Life Years	Salvage Value	Units Produced (Per Hour)
A	200,000	2,500	25	-	5
B	500,000	8,000	15	20,000	X

   The effective annual rate of interest is 10%. Determine X such that the manager is indifferent between the two machines.

7. **[10 pts]** Mr. A sells a stock short at the beginning of the year for 1500. At the end of the year Mr. A is able to buy it for 1200, and delivers it. A margin of 50% was required and was placed into an account that credited interest at 6%. The stock itself paid out a dividend of 70 at the end of the year. What was the yield rate to the short-seller?

8. In each financial instrument, which of the given four items is/are false?
   (a) **[2 pts]** Money Market Funds (MMF)
      i. provide high liquidity and attractive yields;
      ii. contains a variety of short-term, fixed-income securities issued by governments and private firms
      iii. credited rates fluctuate frequently with movements in short-term interest rates
      iv. offers more diversification than what an individual can achieve on their own
   (b) **[2 pts]** Mutual Funds
      i. pooled investment accounts; an investor buys shares in the fund
      ii. rates are guaranteed for a fixed period of time ranging from 30 days to 6 months
      iii. offers more diversification than what an individual can achieve on their own
      iv. usually yields higher interest than ordinary savings account.

(continued on next page)

(c) **[2 pts]** Certificate of Deposits

i. rates are guaranteed for a fixed period of time ranging from 30 days to 6 months

ii. offers more diversification than what an individual can achieve on their own

iii. yield rates are usually more stable than MMF's but less liquid

iv. withdrawal penalties tend to encourage a secondary trading market rather than cashing out

(d) **[2 pts]** Options (Derivative Instrument: its value depends on the marketplace)

i. fixed price at a future date

ii. call option gives the owner the right to sell; put option gives the owner the right to buy

iii. one motivation for buying or selling options is speculation; option prices depend on the value of the underlying asset (leverage)

iv. another motivation (and quite opposite to speculation) is developing hedging strategies to reduce investment risk

END OF EXAM